

# Binary Search Trees

## Part Two

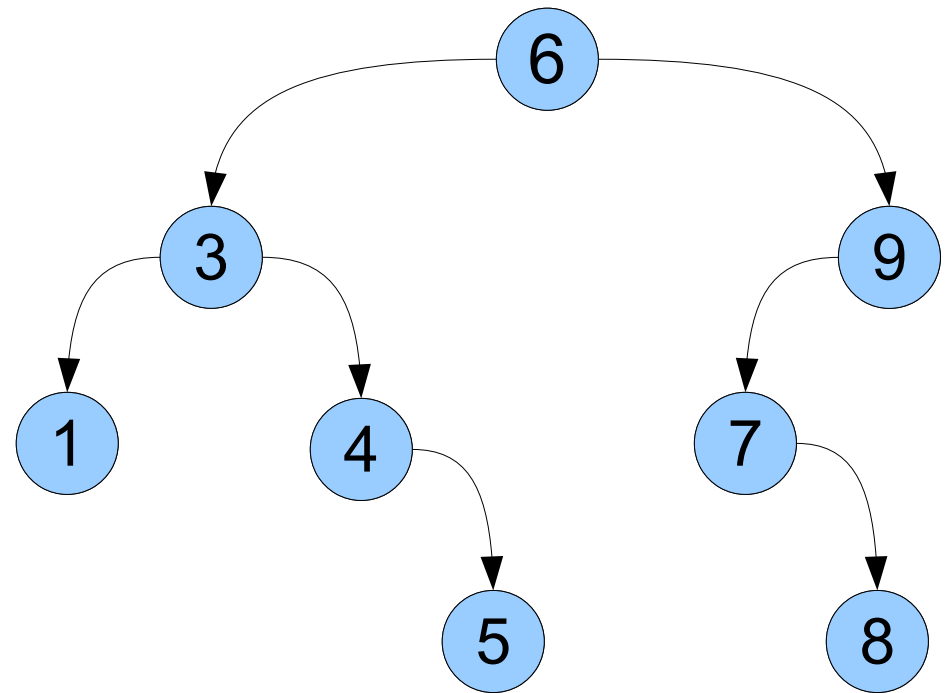
# Outline for Today

- ***Freeing Trees***
  - Cleaning up our messes.
- ***Balanced Trees***
  - How fast are BST operations?
- ***Range Searches***
  - A useful hybrid algorithm.

Recap from Last Time

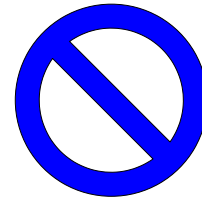
# Binary Search Trees

- The data structure we have just seen is called a **binary search tree** (or **BST**).
- The tree consists of a number of **nodes**, each of which stores a value and has zero, one, or two **children**.
- All values in a node's left subtree are **smaller** than the node's value, and all values in a node's right subtree are **greater** than the node's value.

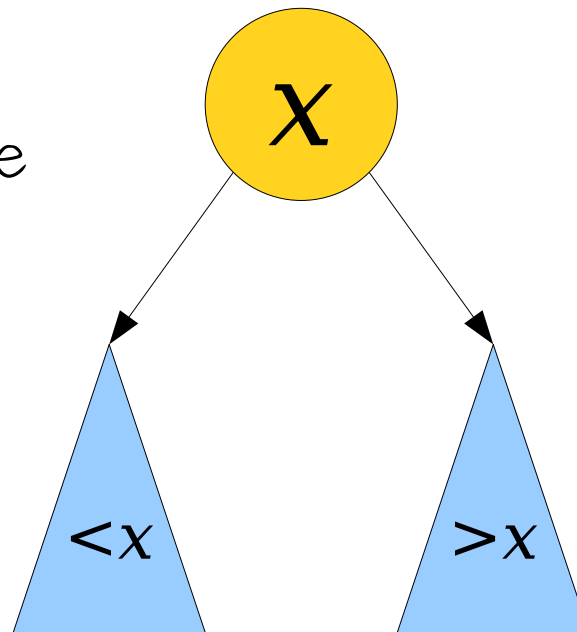


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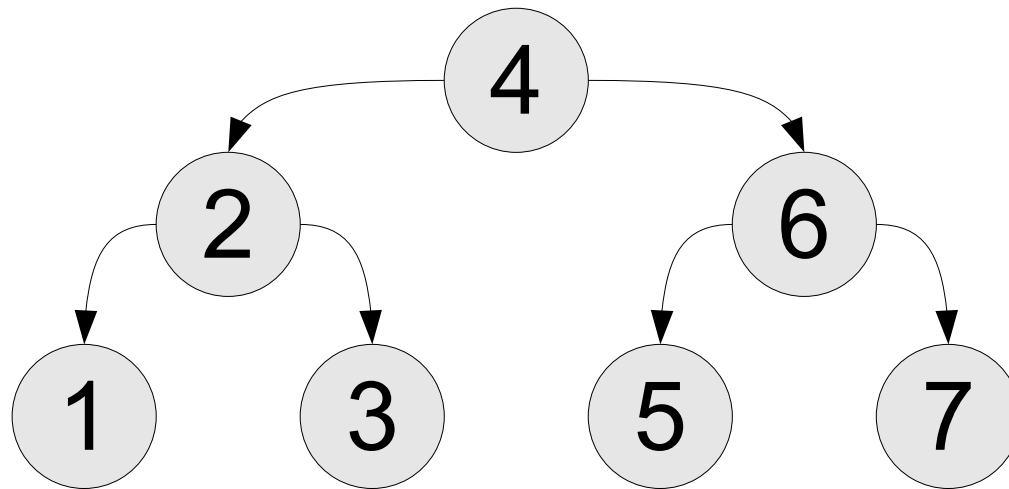
New Stuff!

# Getting Rid of Trees



# Freeing a Tree

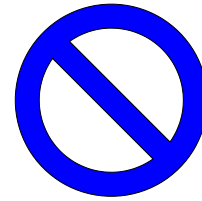
- Once we're done with a tree, we need to free all of its nodes.
- As with a linked list, we have to be careful not to use any nodes after freeing them.



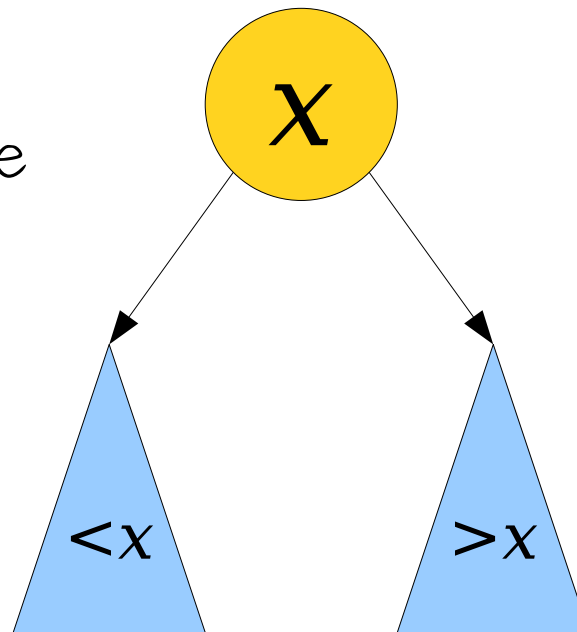


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```
void deleteTree(Node* root) {  
    if (root == nullptr) return;  
  
    delete root;  
    deleteTree(root->left);  
    deleteTree(root->right);  
}
```

A

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void deleteTree(Node* root) {  
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```

B

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}
```

C

```
void deleteTree(Node* root) {  
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}
```

D

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void deleteTree(Node* root) {  
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    delete root;  
}
```

E

```
void deleteTree(Node* root) {  
    if (root == nullptr) return;  
  
    deleteTree(root->right);  
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}
```

F

Which of these options work?  
Answer at <https://pollev.com/cs106bwin23>

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```



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# Postorder Traversals

- The particular recursive pattern we just saw is called a ***postorder traversal*** of a binary tree.
- Specifically:
  - Recursively visit all the nodes in the two subtrees, in whichever order you'd like.
  - Visit the node itself.
- This contrasts with the ***inorder traversal*** we used to print the contents of a BST.
  - That's where we recursively visit the left subtree, then the node itself, then the right subtree.

# Tree Efficiency

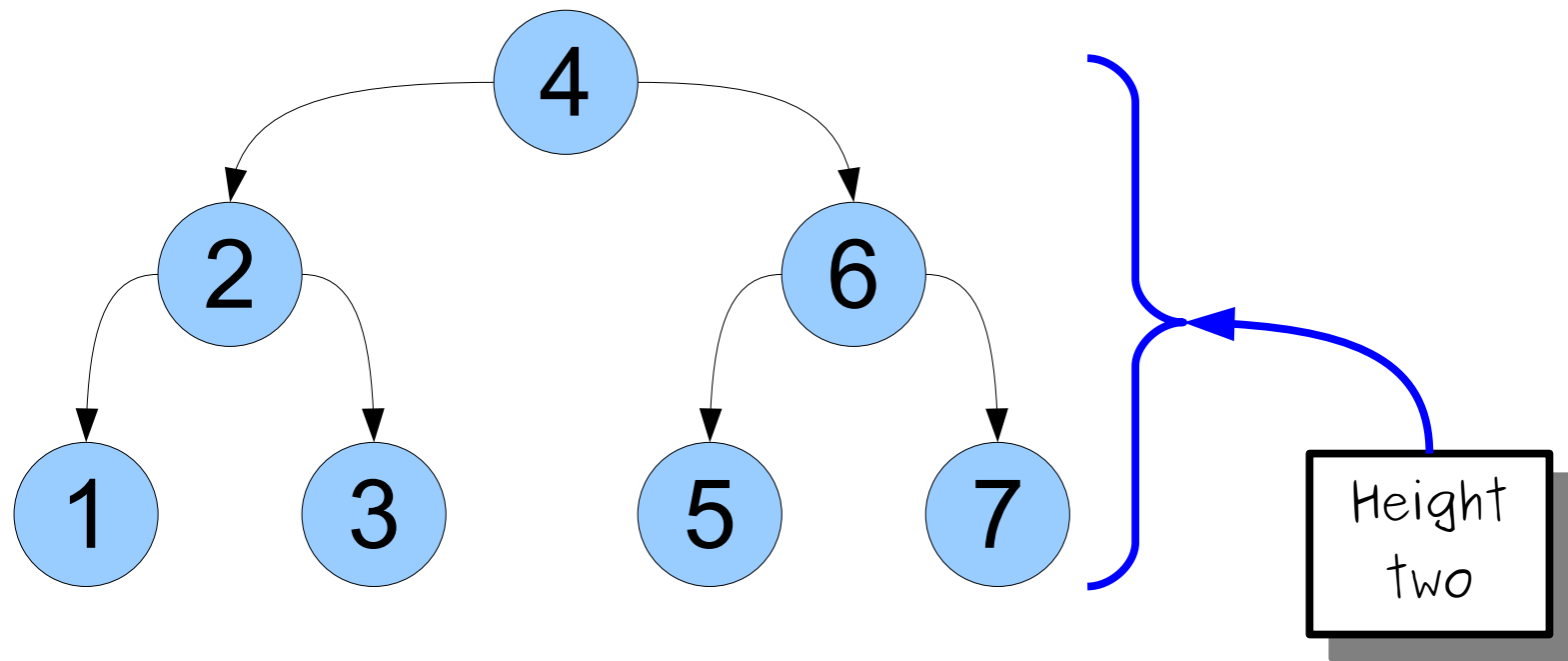


How fast are BST lookups?

How fast are BST insertions?

# Tree Terminology

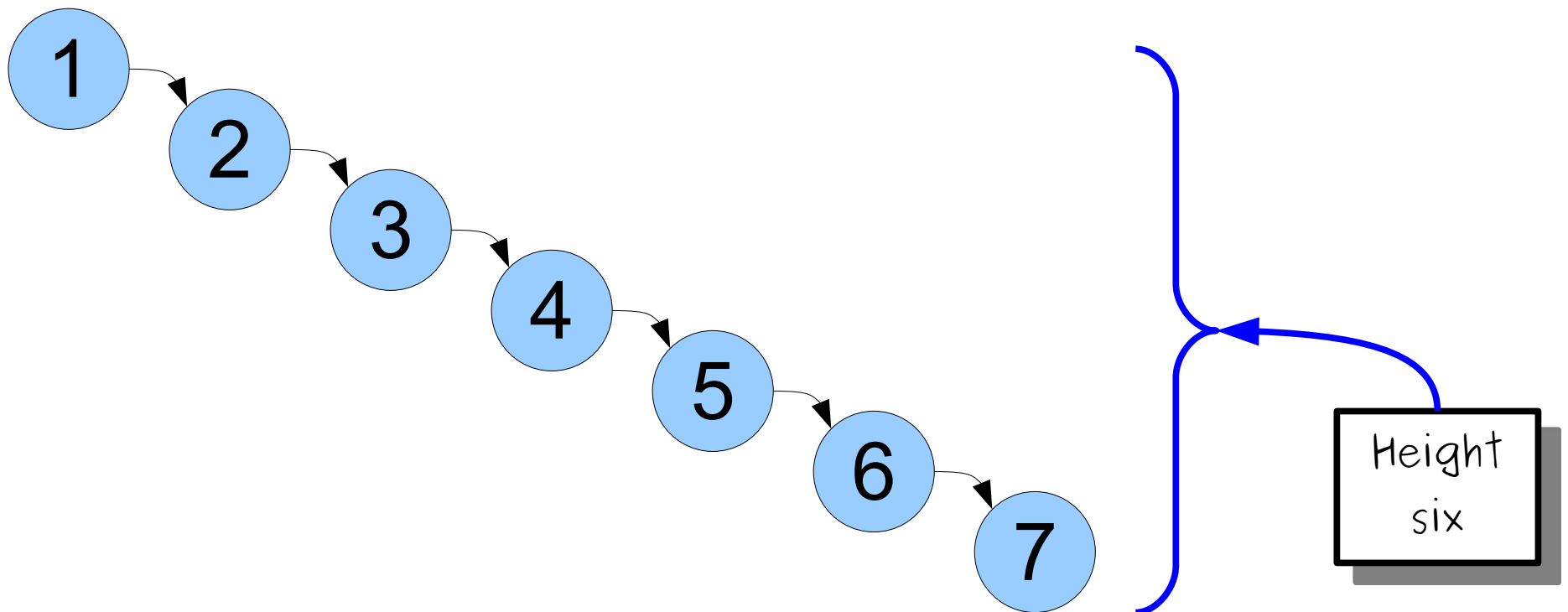
- The ***height*** of a tree is the number of links in the longest path from the root to a leaf.





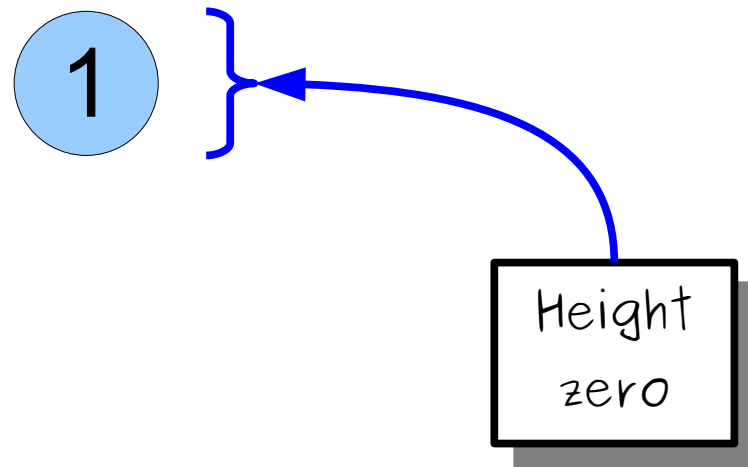
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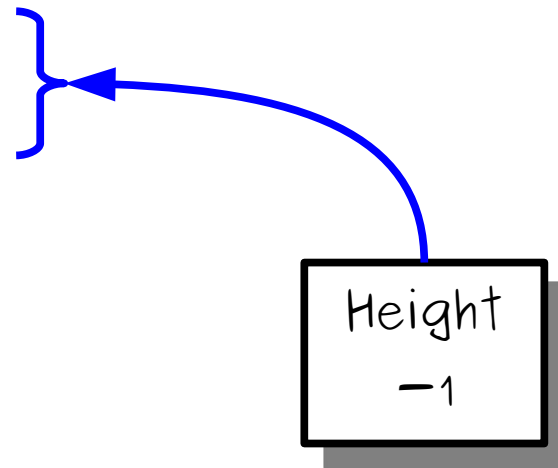
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- The *height* of a tree is the number of links in the longest path from the root to a leaf.
- By convention, an empty tree has height -1.



# Building a BST

- First, draw the BST formed by inserting the values 1, 3, 5, 7, 2, 4, 6 into an empty tree.
- Then draw what you get if you insert the values 4, 6, 5, 2, 1, 7, and 3 into an empty tree.

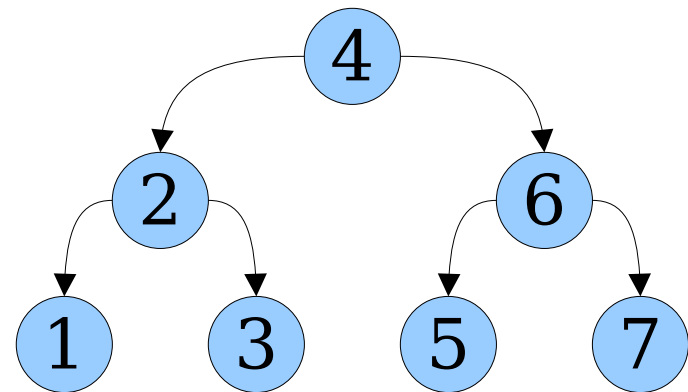
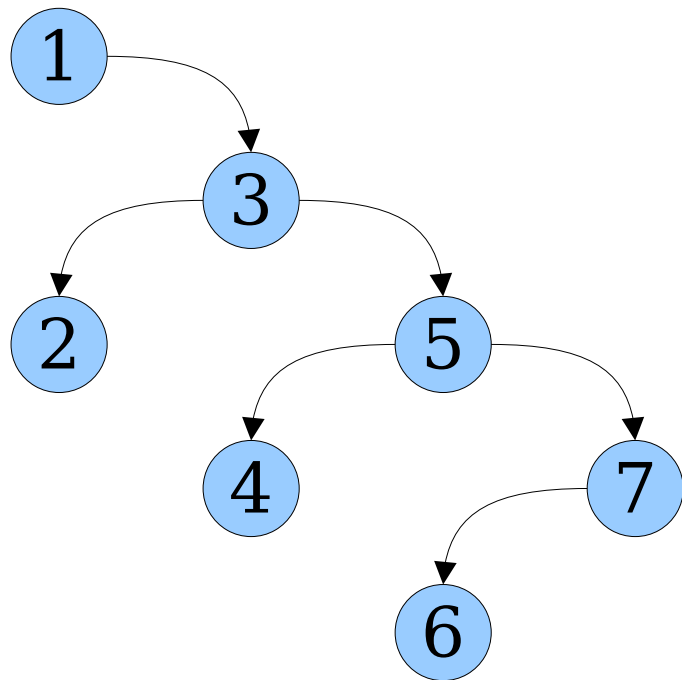
Draw these trees. What is the height of each tree?

Answer at

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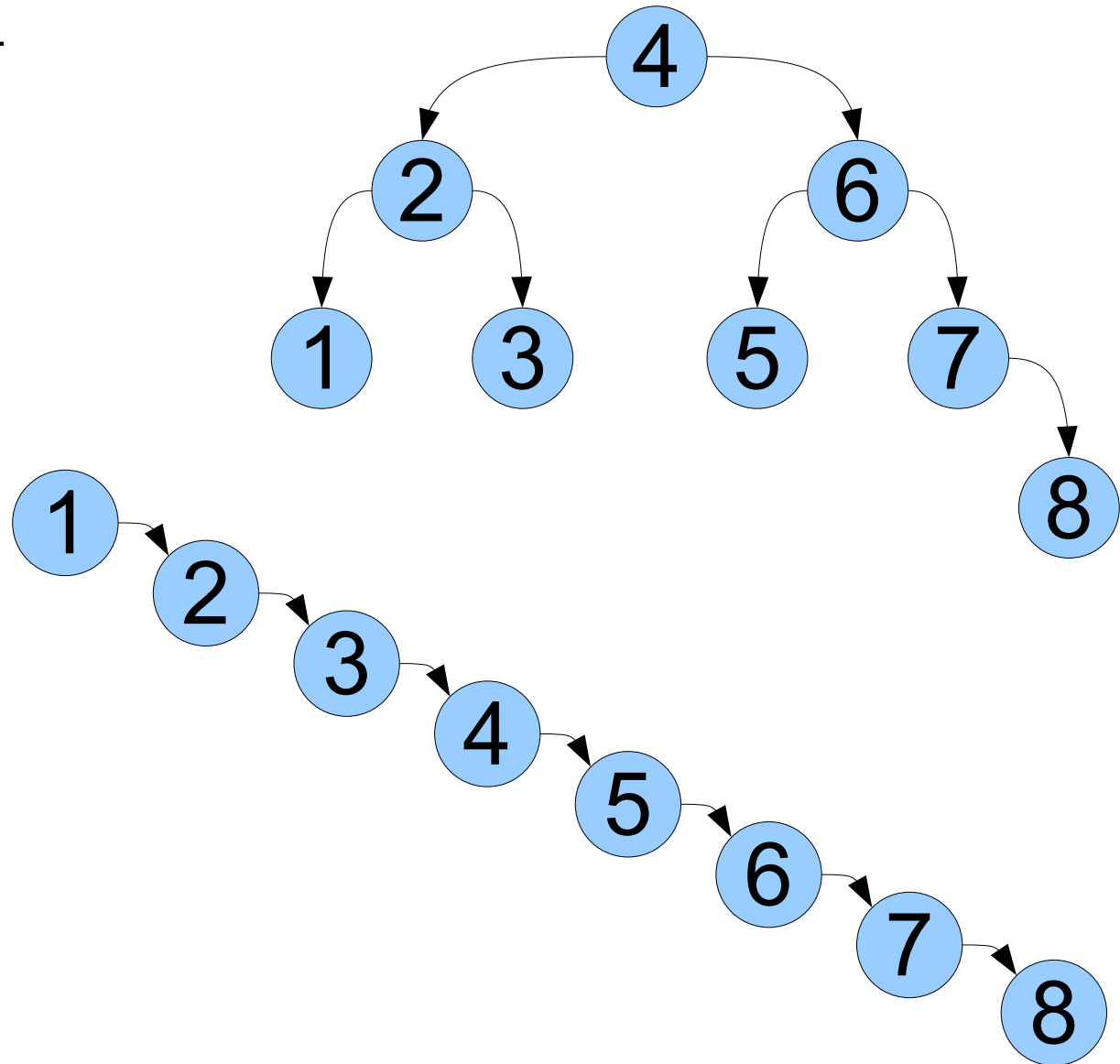
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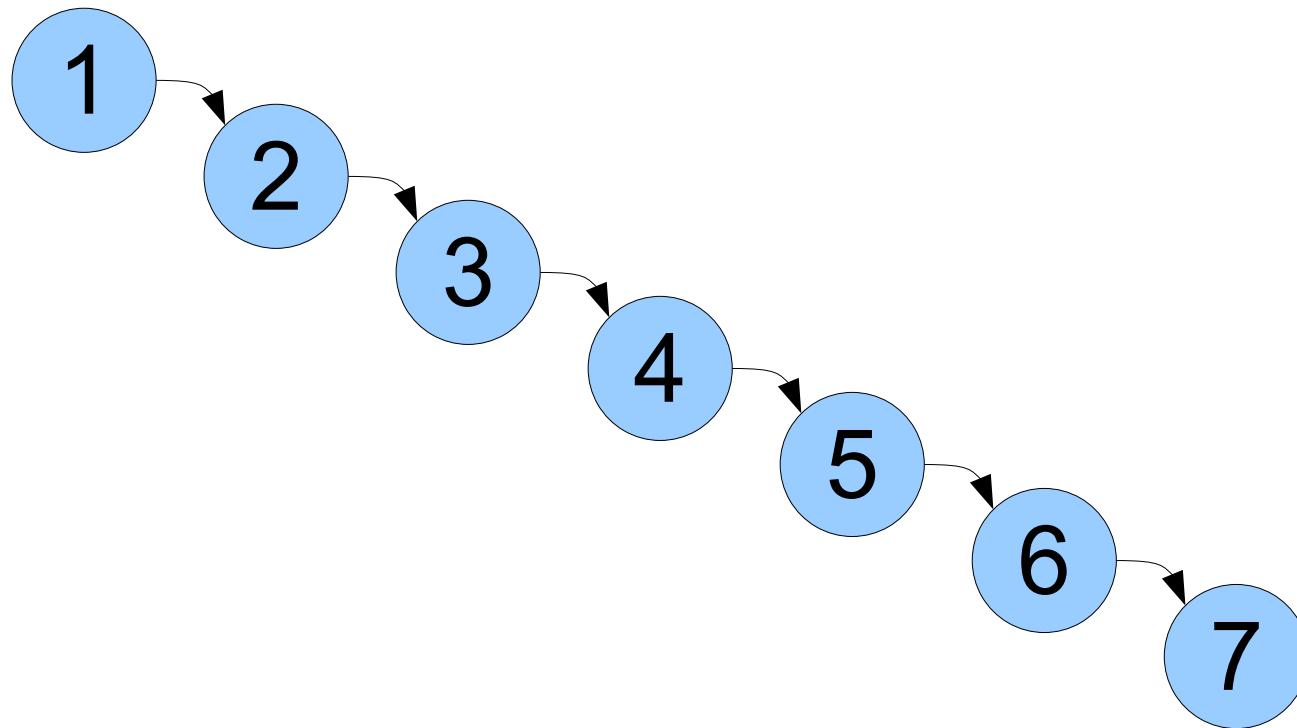
# Efficiency Questions

- The time to add an element to a BST (or look up an element in a BST) depends on the height of the tree.
- The runtime is  **$O(h)$** , where  $h$  is the height of the tree.



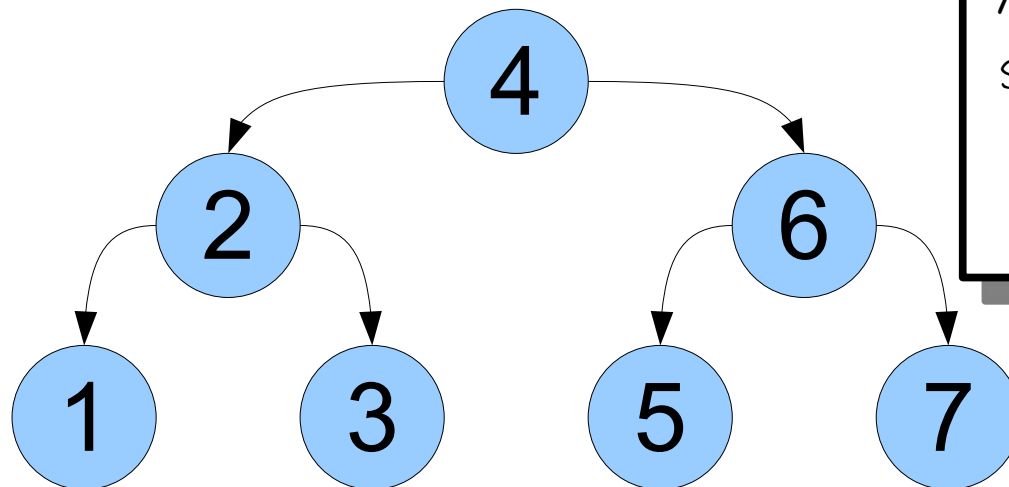
# Tree Heights

- What is the maximum and minimum possible height of a tree with  $n$  nodes?
- Maximum height: all nodes in a chain. Height is  $O(n)$ .



# Tree Heights

- What is the maximum and minimum possible height of a tree with  $n$  nodes?
- Maximum height: all nodes in a chain. Height is  $O(n)$ .
- Minimum height: tree is as complete as possible. Height is  $O(\log n)$ .



You can only double something  $O(\log n)$  times before it exceeds  $n$ .



# Balanced Trees

- A binary search tree is called ***balanced*** if its height is  $O(\log n)$ , where  $n$  is the number of nodes in the tree.
- Balanced trees are extremely efficient:
  - Lookups take time  $O(\log n)$ .
  - Insertions take time  $O(\log n)$ .
  - Deletions take time  $O(\log n)$ .
- ***Question:*** How do you balance a tree?

# Balanced Trees

- A ***self-balancing tree*** is a BST that reshapes itself on insertions and deletions to stay balanced.
- There are many strategies for doing this. They're beautiful. They're clever. And they're beyond the scope of CS106B.
- Some suggested topics to read up on, if you're curious:
  - Red/black trees (take CS161 or CS166!)
  - AVL trees (covered in the textbook.)
  - Splay trees (trees that reshape on lookups.)
  - Scapegoat trees (yes, that's what they're called.)
  - Treaps (half binary heap, half binary search tree!)

What if you do no balancing at all?

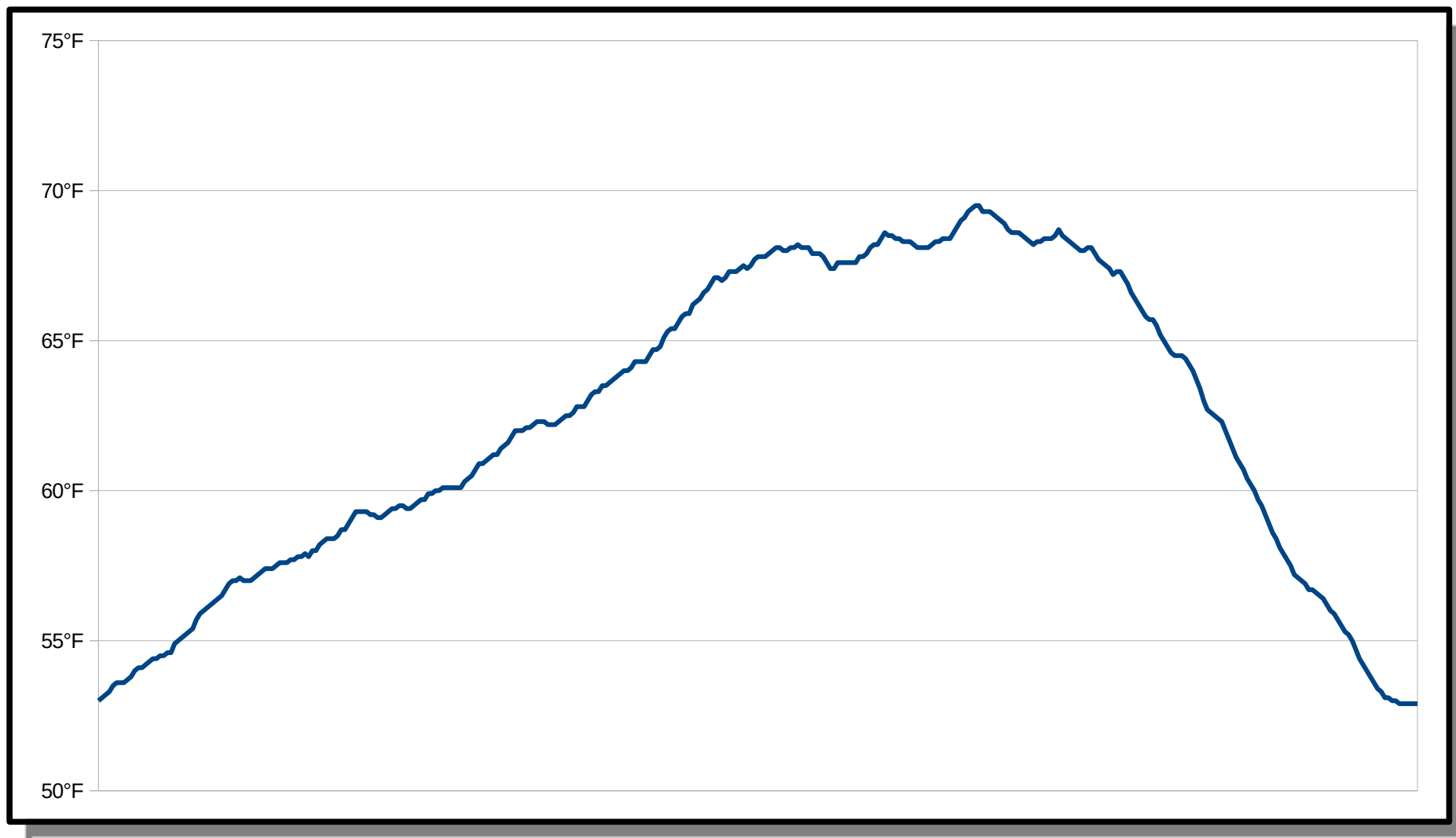
# A Tale of Two Trees

- We have a thermometer that gives a temperature reading at 4PM each day. We insert the temperature readings into a BST each day, starting on January 1 and ending on December 31.
- There's a marathon race. We insert the names of the athletes into a BST as they cross the finish line.

Which BST will be more balanced?  
Which BST will be less balanced?  
Why?

Answer at

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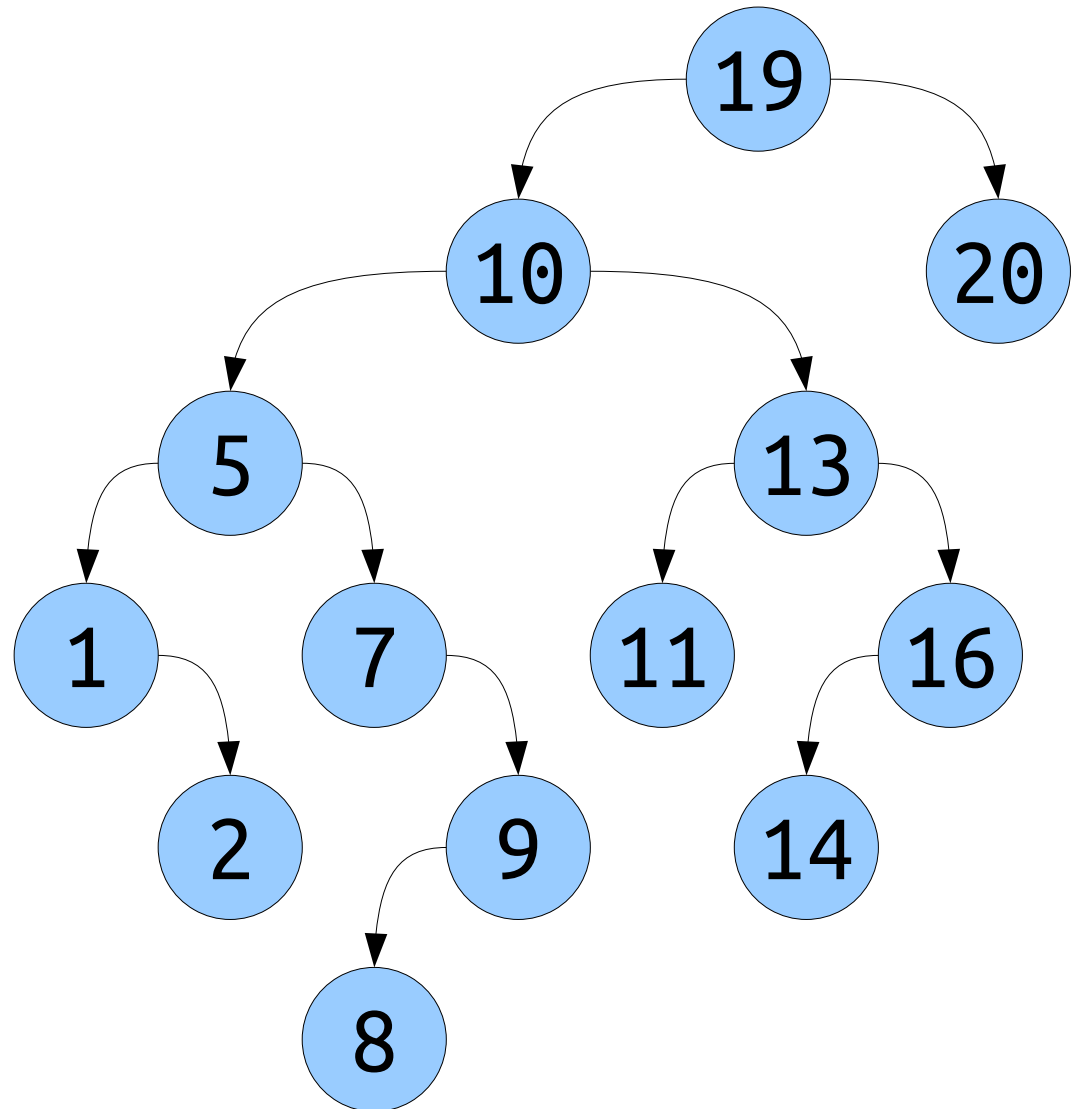


Temperature readings, inserted  
daily at 4PM, from January 1 to  
December 31.

*(Data source: NOAA: SFO readings from Jan 1 - Dec 31 2010)*

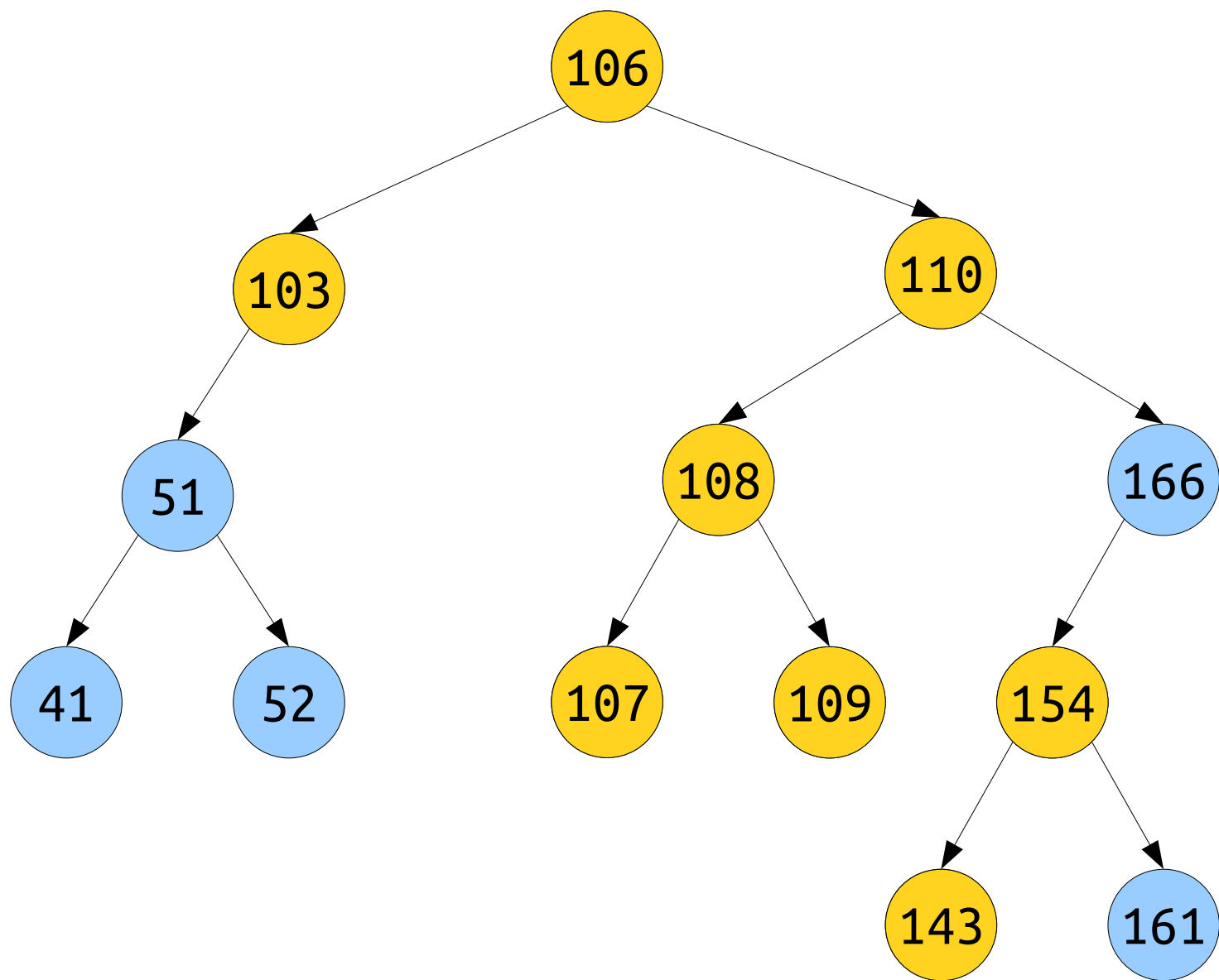
# Balanced Trees

- **Theorem:** If you start with an empty tree and add in random values, then, with high probability, the tree is balanced.
- **Proof:** Take CS161!
- **Takeaway:** If you're adding elements to a BST and their values are actually random, then your tree is likely to be balanced.



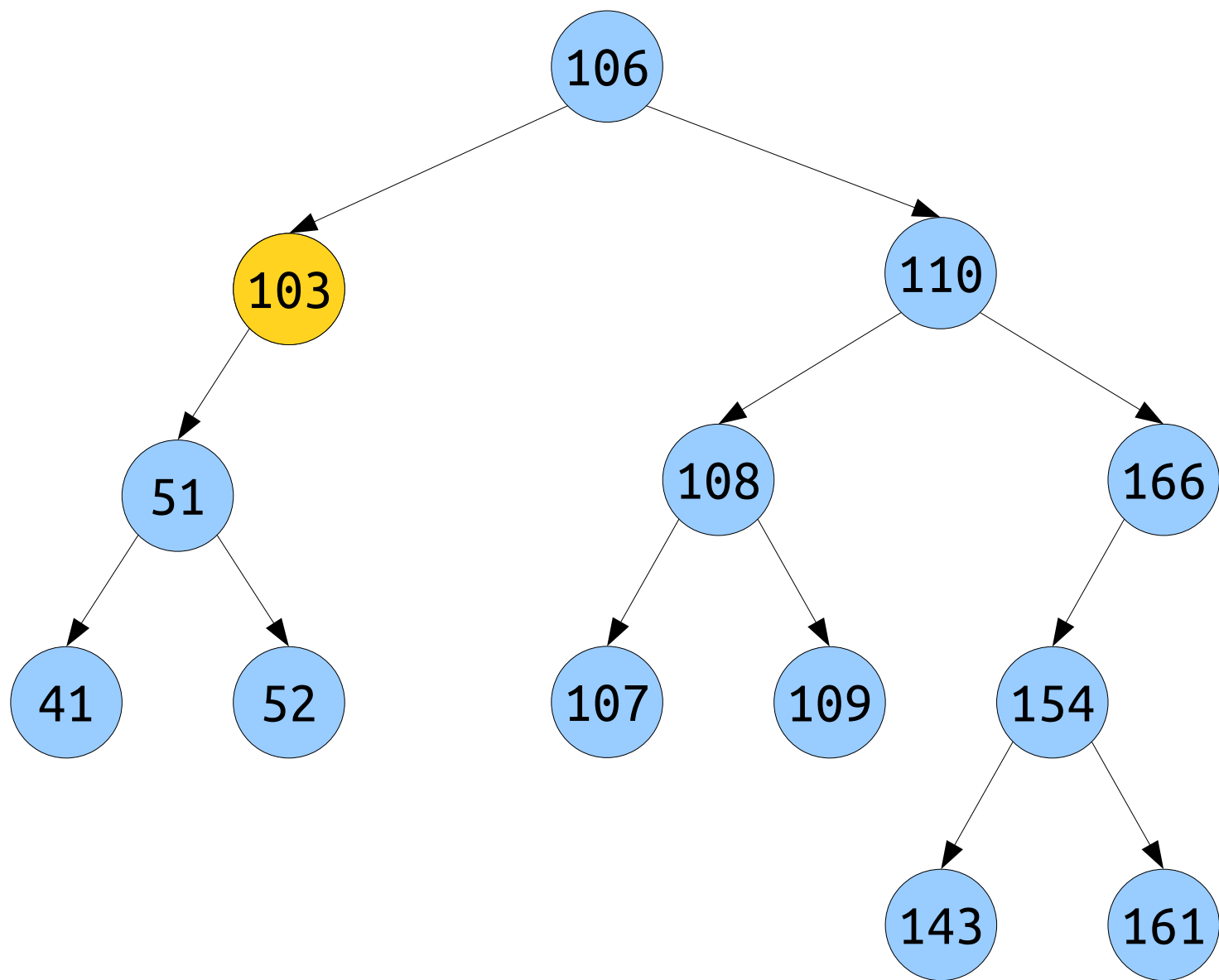
# Range Searches



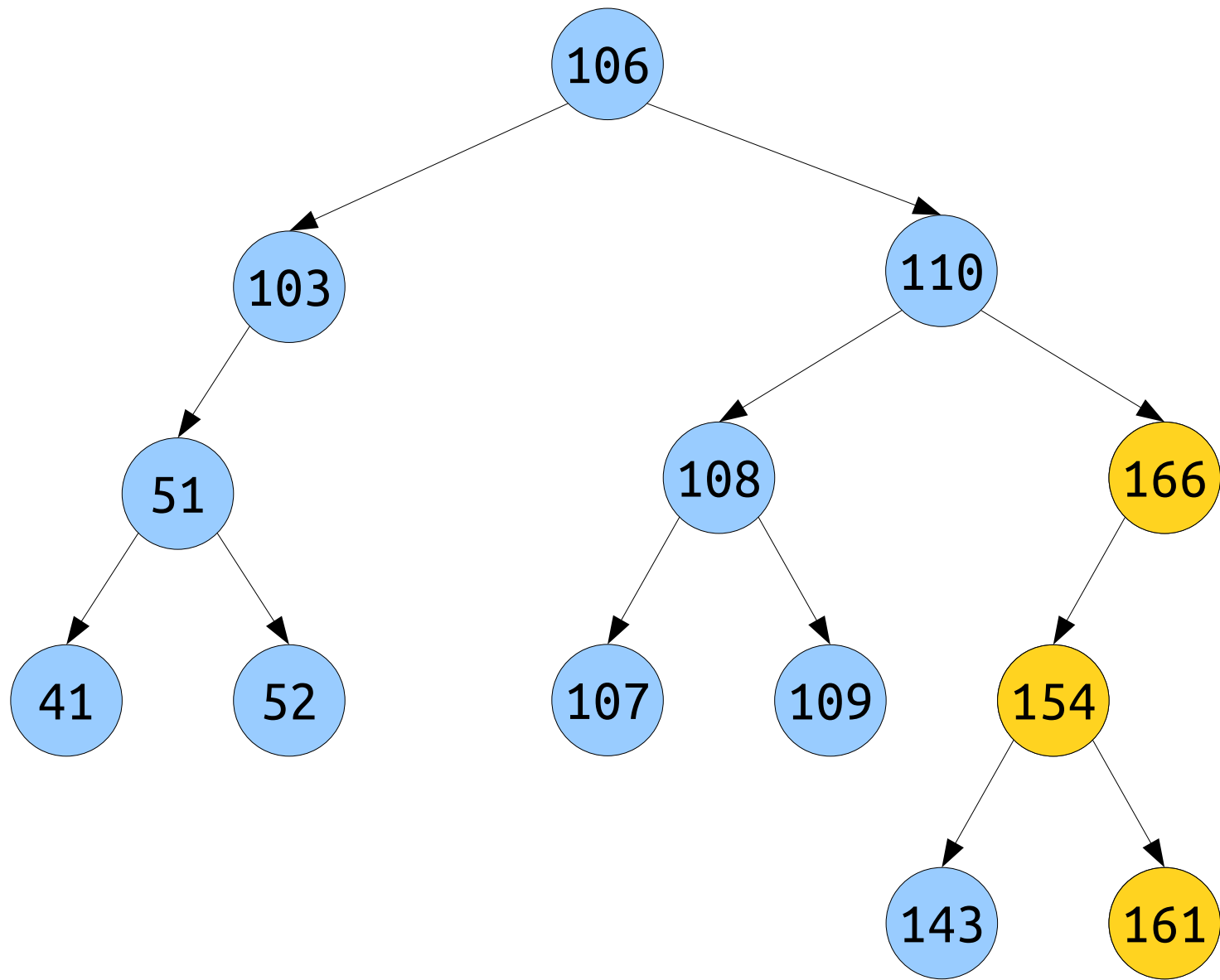


Find all elements in this tree in the range **[103, 154]**.

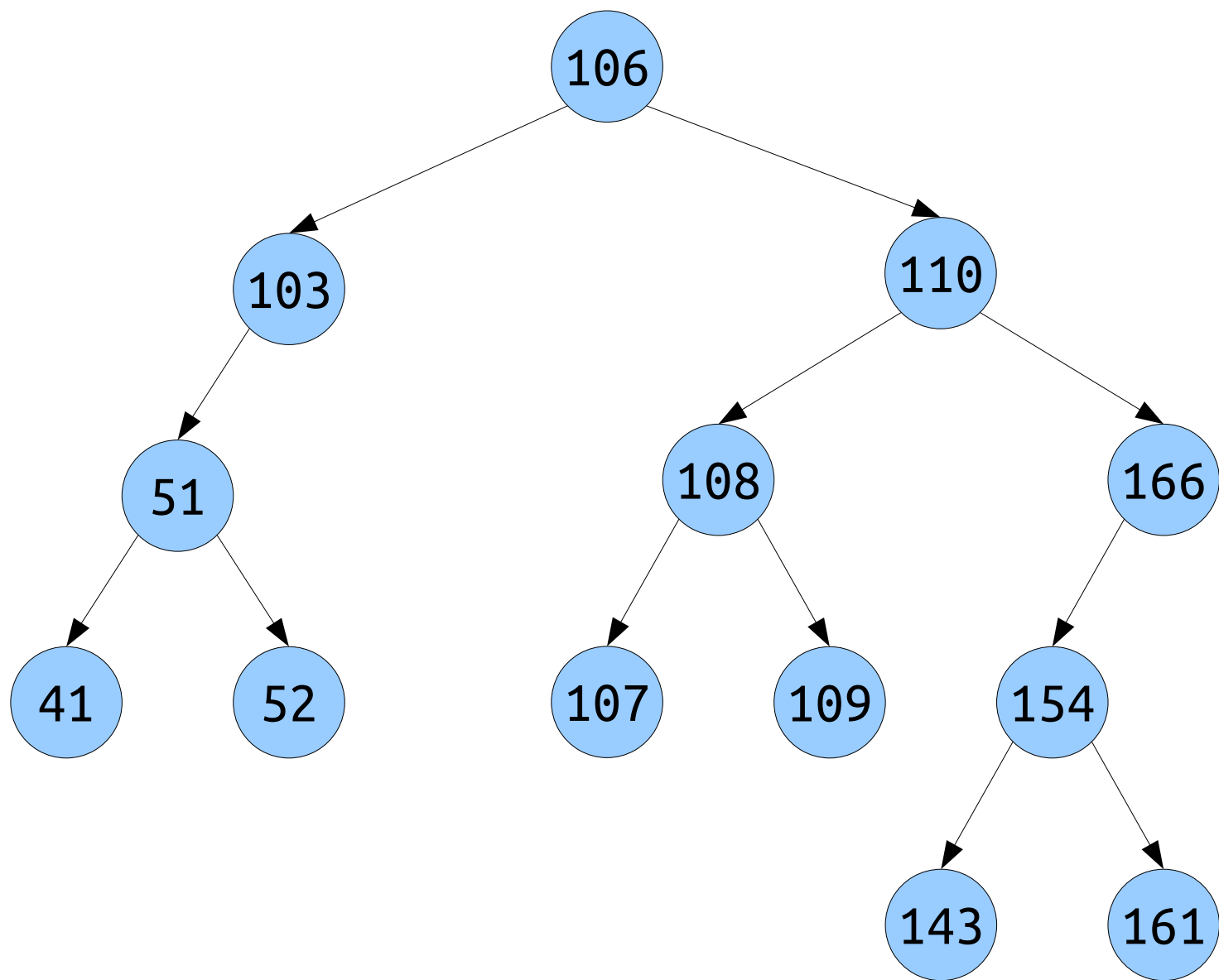




Find all elements in this tree in the range **[99, 105]**.



Find all elements in this tree in the range **[150, 170]**.



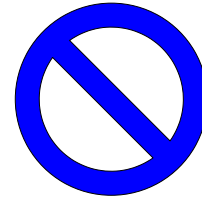
Find all elements in this tree in the range **[137, 138]**.

# Range Searches

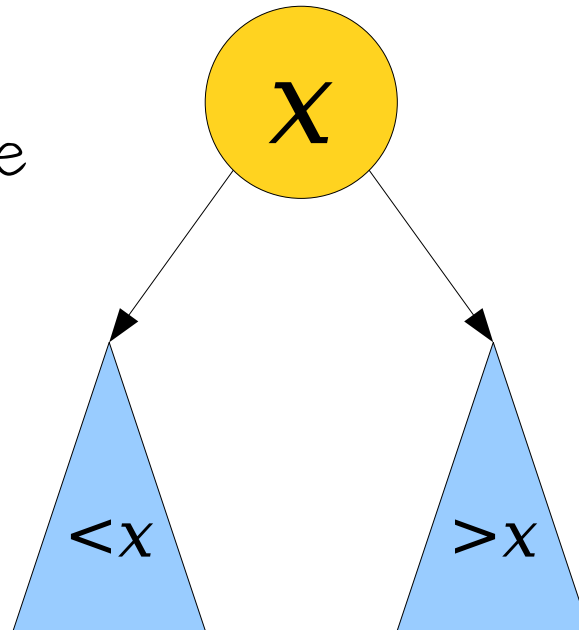
- We can use BSTs to do *range searches*, in which we find all values in the BST within some range.
- For example:
  - If the values in the BST are dates, we can find all events that occurred within some time window.
  - If the values in the BST are number of diagnostic scans ordered, we can find all doctors who order a disproportionate number of scans.

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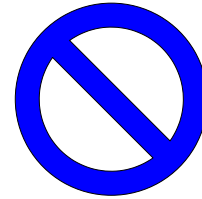
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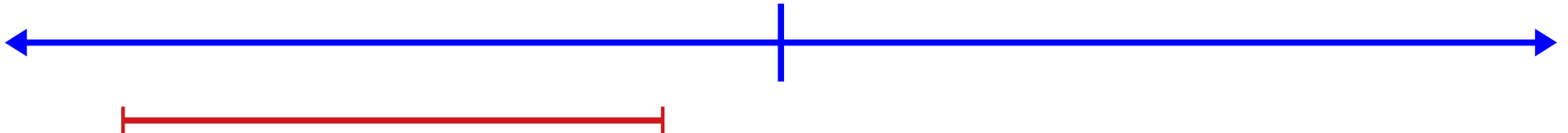
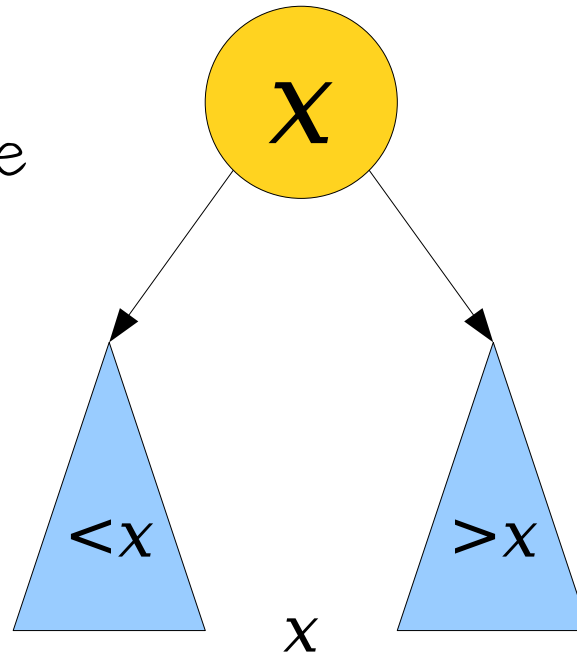
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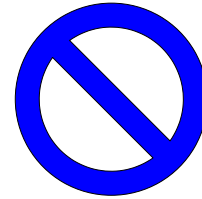
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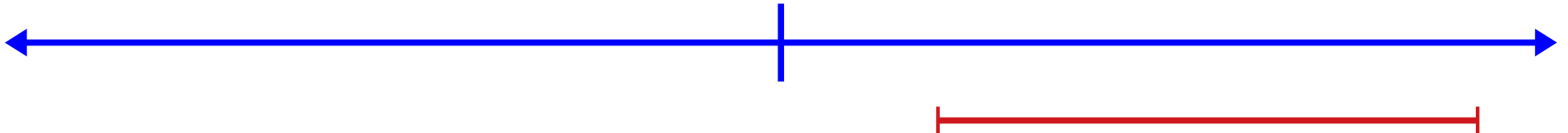
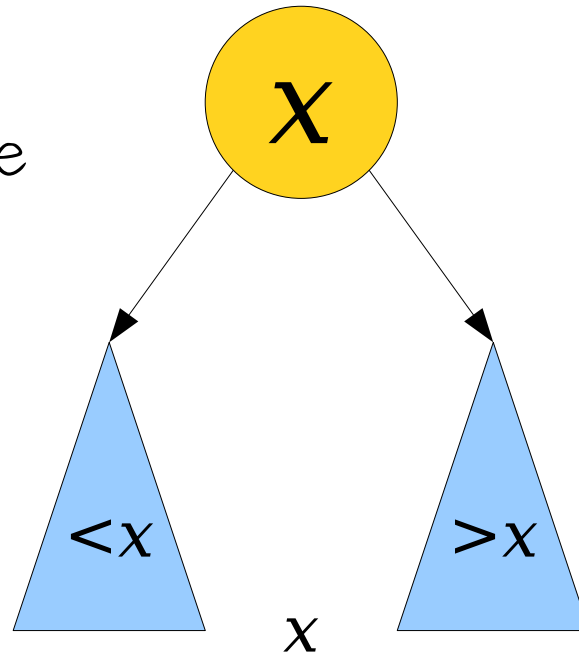
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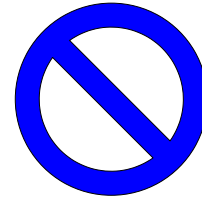
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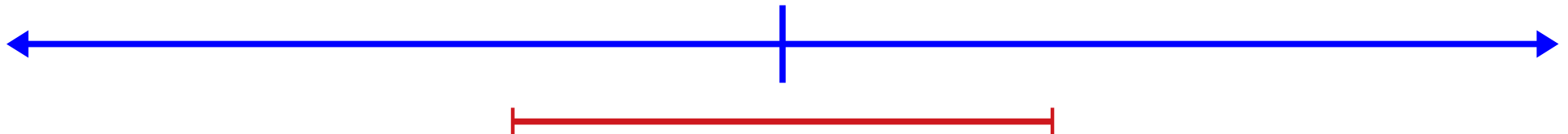
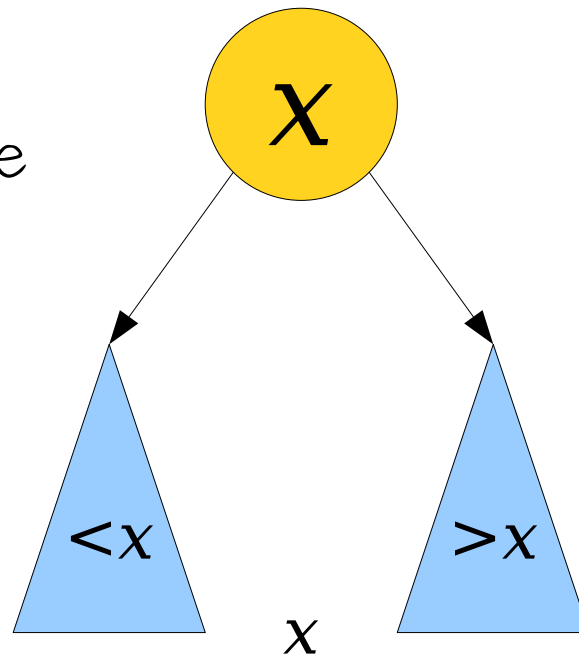
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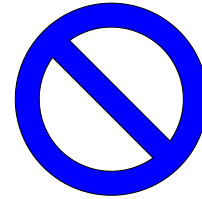
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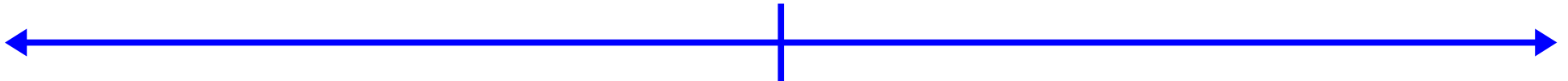
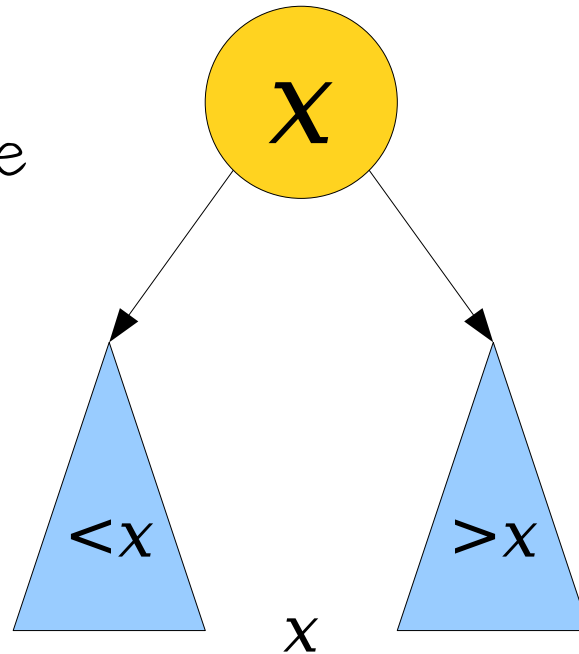
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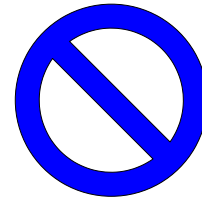
# Range Searches

- A hybrid between an inorder traversal and a regular BST lookup!
- The idea:
  - If the node is in the range being searched, add it to the result.
  - Recursively explore each subtree that could potentially overlap with the range.
- ***Fun fact:*** The runtime of a range search is  $O(h + z)$ , where  $h$  is the height of the tree and  $z$  is the number of items in the range. Come chat with me after class if you're curious why this is!

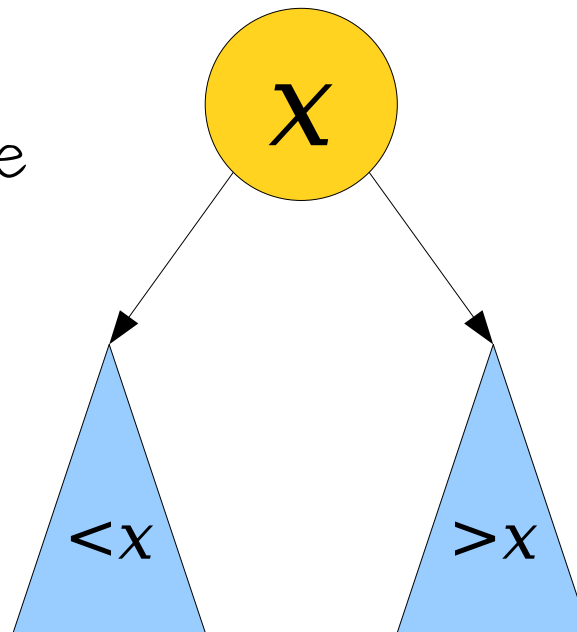
To Summarize:

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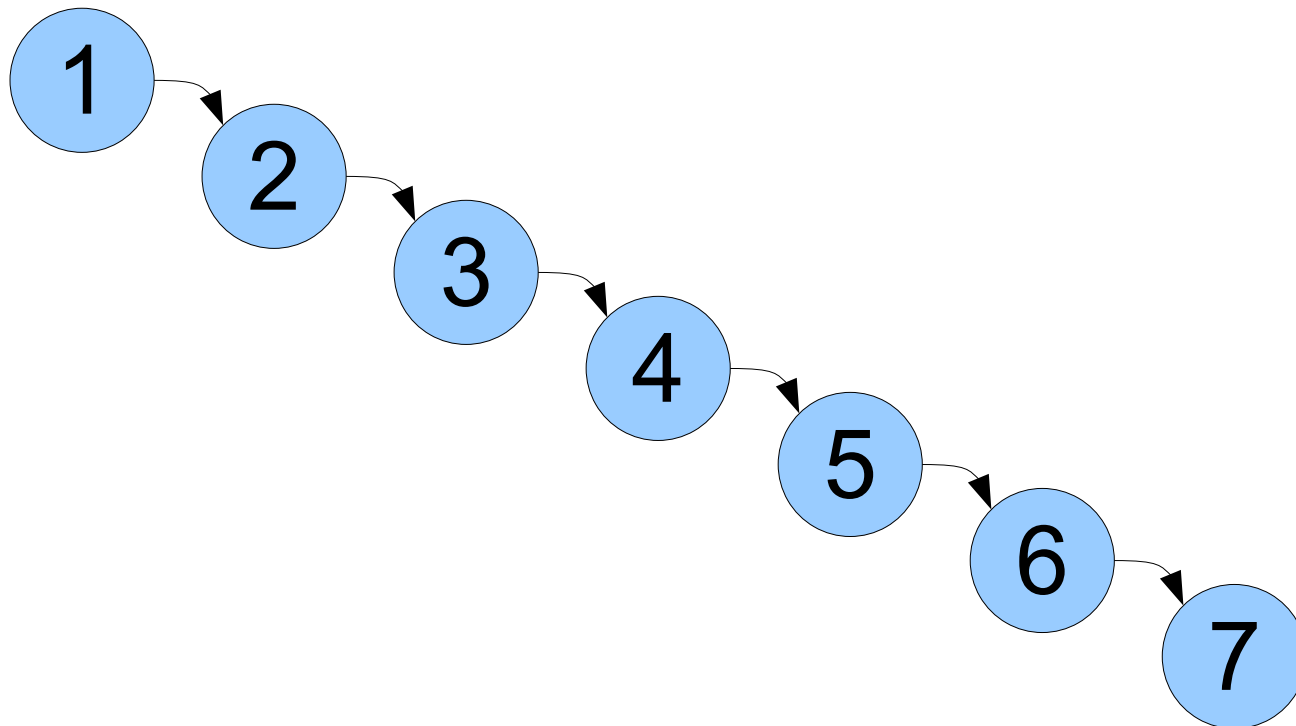
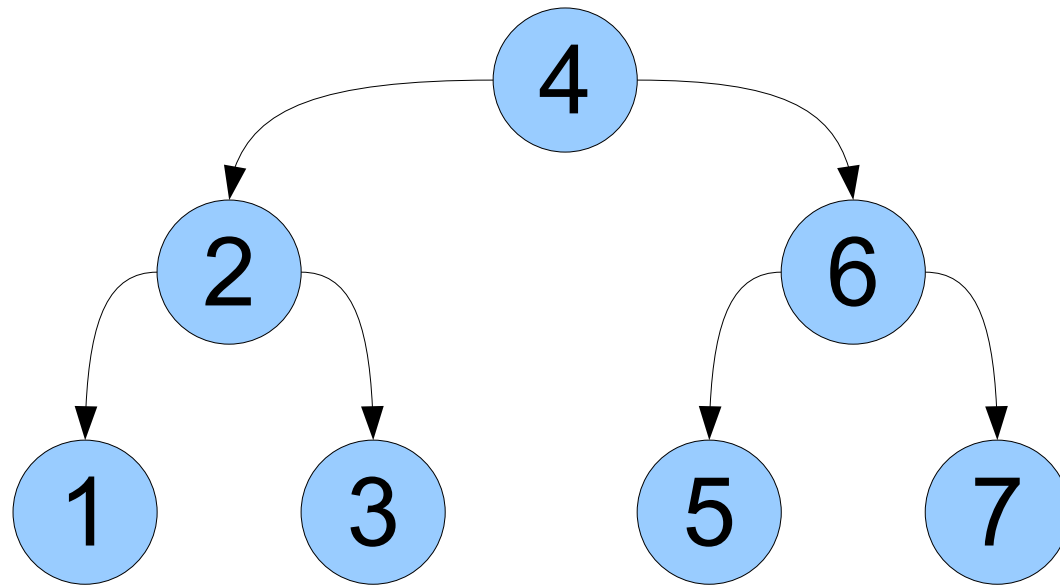


... and whose right  
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```
struct Node {  
    Type value;  
    Node* left; // Smaller values  
    Node* right; // Bigger values  
};
```

```
bool contains(Node* root, const string& key) {  
    if (root == nullptr) return false;  
    else if (key == root->value) return true;  
    else if (key < root->value) return contains(root->left, key);  
    else return contains(root->right, key);  
}
```

```
void insert(Node*& root, const string& key) {  
    if (root == nullptr) {  
        root = new Node;  
        node->value = key;  
        node->left = node->right = nullptr;  
    } else if (key < root->value) {  
        insert(root->left, key);  
    } else if (key > root->value) {  
        insert(root->right, key);  
    } else {  
        // Already here!  
    }  
}
```



```
void printContentsOf(Node* root) {  
    if (root == nullptr) return;  
  
    printContentsOf(root->left);  
    cout << root->value << endl;  
    printContentsOf(root->right);  
}
```

```
void deleteTree(Node* root) {  
    if (root == nullptr) return;  
  
    deleteTree(root->left);  
    deleteTree(root->right);  
    delete root;  
}
```



```
void printInRange(Node* tree, const string& low, const string& high) {  
    if (tree == nullptr) return;  
  
    if (high < tree->value) {  
        printInRange(tree->left, low, high);  
    } else if (low > tree->value) {  
        printInRange(tree->right, low, high);  
    } else {  
        printInRange(tree->left, low, high);  
        cout << tree->value << endl;  
        printInRange(tree->right, low, high);  
    }  
}
```

# Your Action Items

- ***Read Chapter 16.1 - 16.2.***
  - All about BSTs!
- ***Work on Assignment 7.***
  - If you are following our timetable, you'll have finished the labyrinth and doubly-linked list warmups and should be in the middle of Particle Systems now.
  - Remember that you *can* use late days on this assignment and *cannot* use them on Assignment 8. Plan accordingly.
    - Don't use late days unnecessarily; that eats into your time for A8.
    - Don't save your late days "just in case" you need them on A8, since you can't use them there.
  - Need help? Have questions? Come talk to us in LaIR or during office hours.

# Next Time

- ***Other Binary Trees***
  - BSTs are wonderful, but other tree structures with similar shapes exist.
- ***Huffman Coding***
  - Practical data compression – with trees!