

Binary Search Trees

Part Two

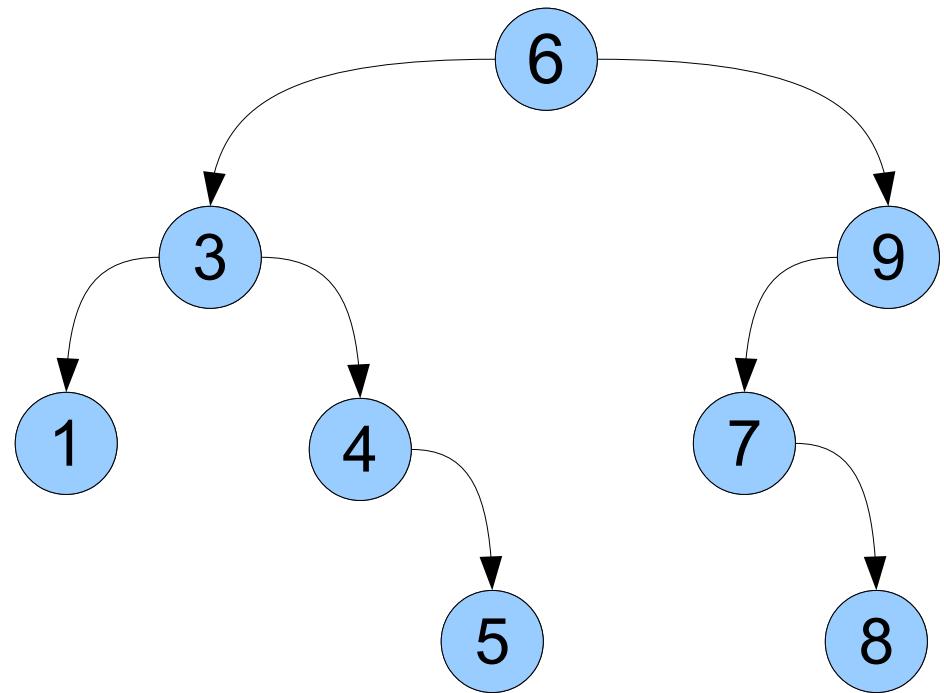
Outline for Today

- ***Freeing Trees***
 - Cleaning up our messes.
- ***Balanced Trees***
 - How fast are BST operations?
- ***Range Searches***
 - A useful hybrid algorithm.

Recap from Last Time

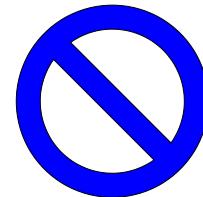
Binary Search Trees

- The data structure we have just seen is called a ***binary search tree*** (or ***BST***).
- The tree consists of a number of ***nodes***, each of which stores a value and has zero, one, or two ***children***.
- All values in a node's left subtree are ***smaller*** than the node's value, and all values in a node's right subtree are ***greater*** than the node's value.



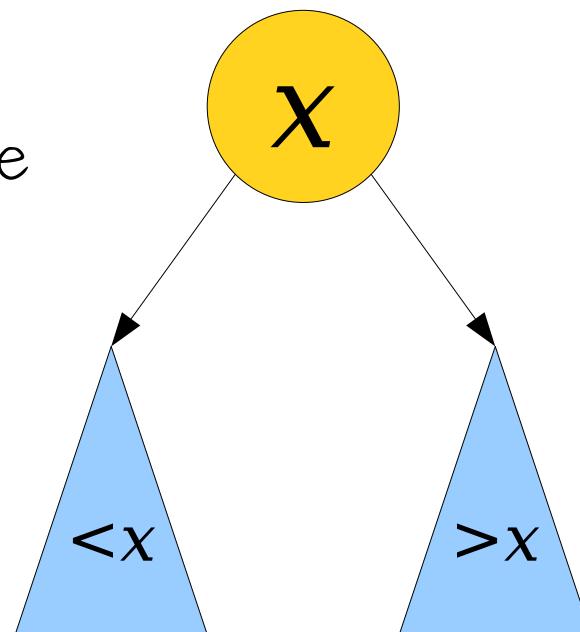
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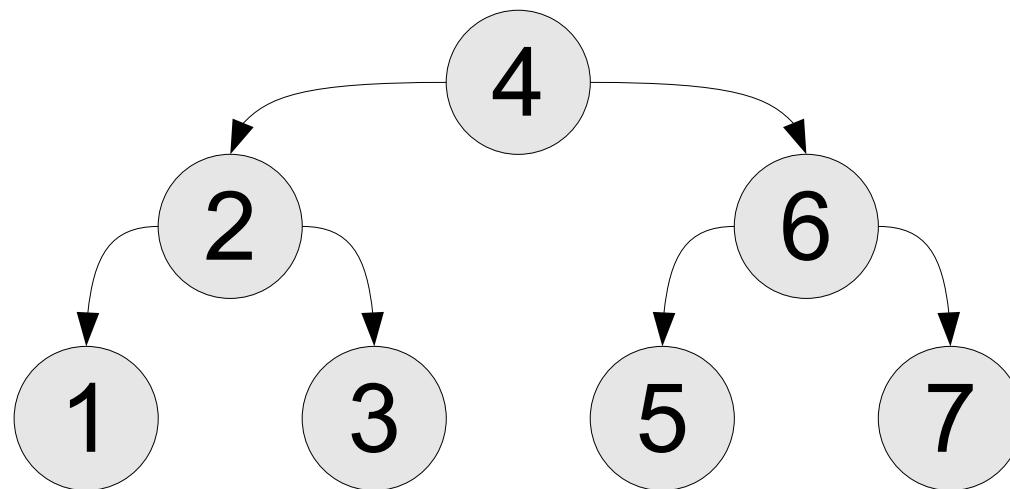
New Stuff!

Getting Rid of Trees



Freeing a Tree

- Once we're done with a tree, we need to free all of its nodes.
- As with a linked list, we have to be careful not to use any nodes after freeing them.



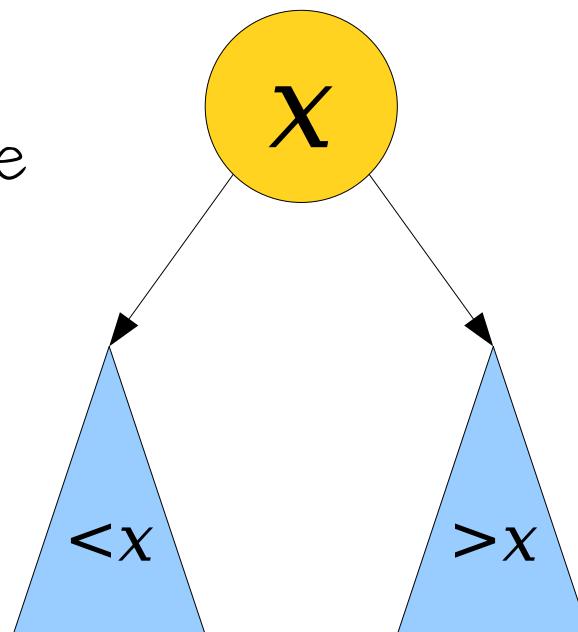
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```
void deleteTree(Node* root) {  
    if (root == nullptr) return;  
  
    delete root;  
    deleteTree(root->left);  
    deleteTree(root->right);  
}
```

A

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void deleteTree(Node* root) {  
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B

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void deleteTree(Node* root) {  
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    deleteTree(root->right);  
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```

C

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void deleteTree(Node* root) {  
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```

E

```
void deleteTree(Node* root) {  
    if (root == nullptr) return;  
  
    deleteTree(root->right);  
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}
```

F

Which of these options work?

Answer at <https://pollev.com/cs106bwin23>

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Which of these options work?

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Postorder Traversals

- The particular recursive pattern we just saw is called a ***postorder traversal*** of a binary tree.
- Specifically:
 - Recursively visit all the nodes in the two subtrees, in whichever order you'd like.
 - Visit the node itself.
- This contrasts with the ***inorder traversal*** we used to print the contents of a BST.
 - That's where we recursively visit the left subtree, then the node itself, then the right subtree.

Tree Efficiency

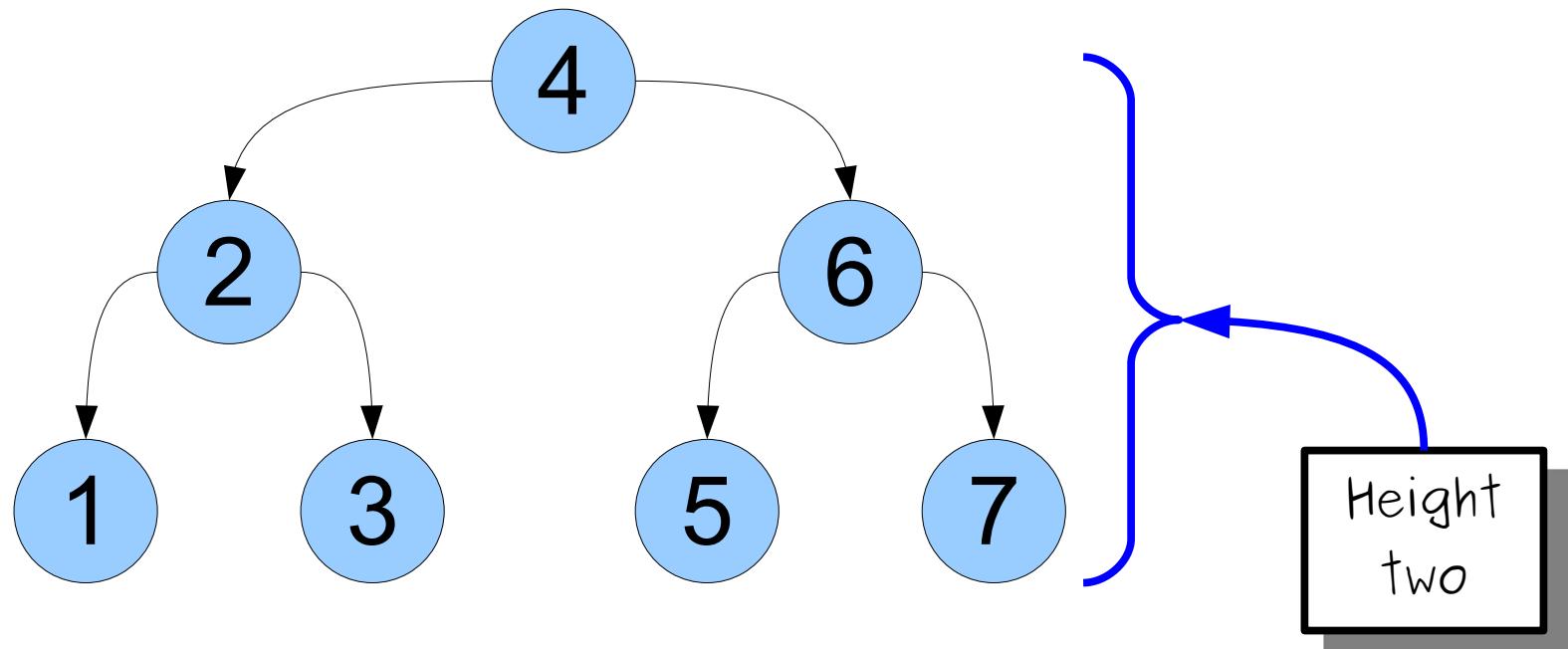


How fast are BST lookups?

How fast are BST insertions?

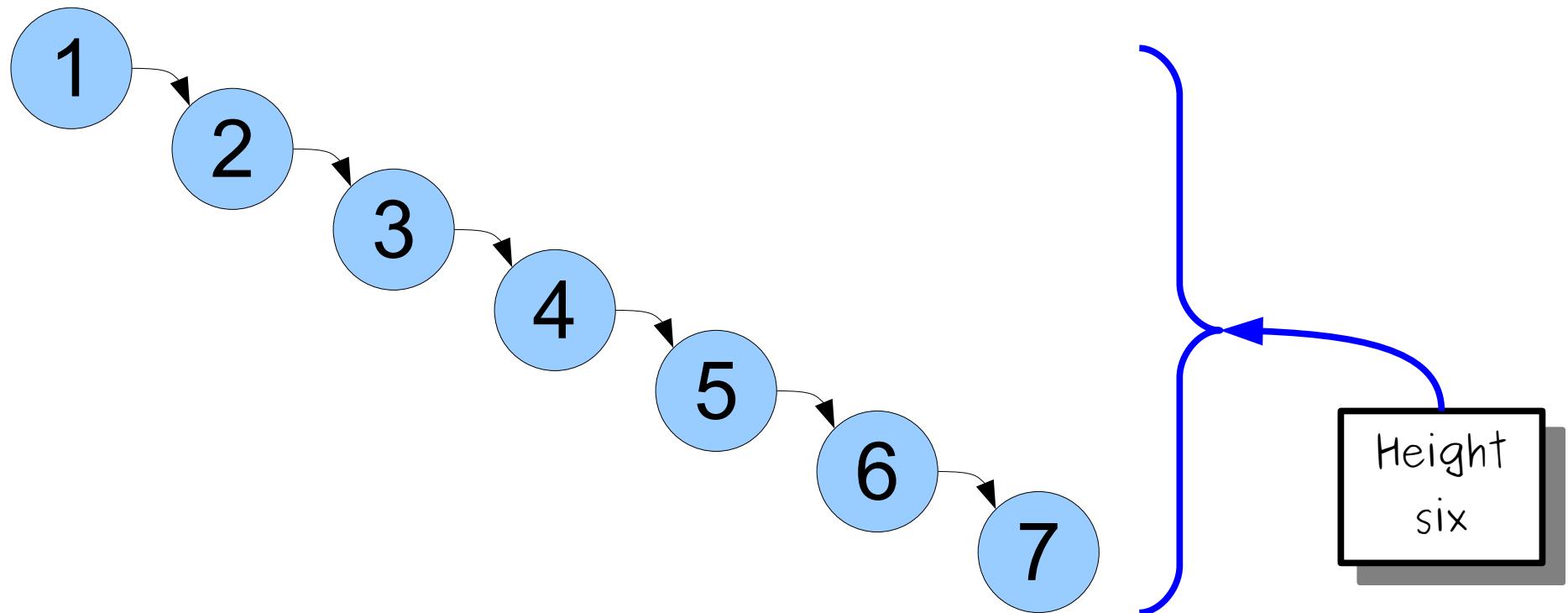
Tree Terminology

- The **height** of a tree is the number of links in the longest path from the root to a leaf.



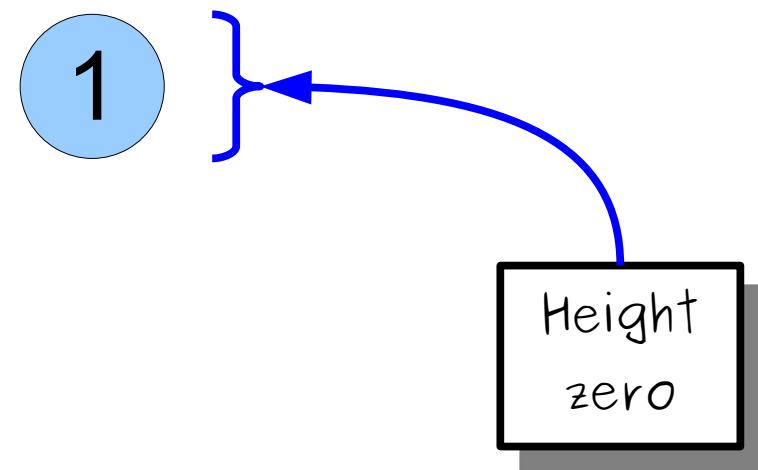
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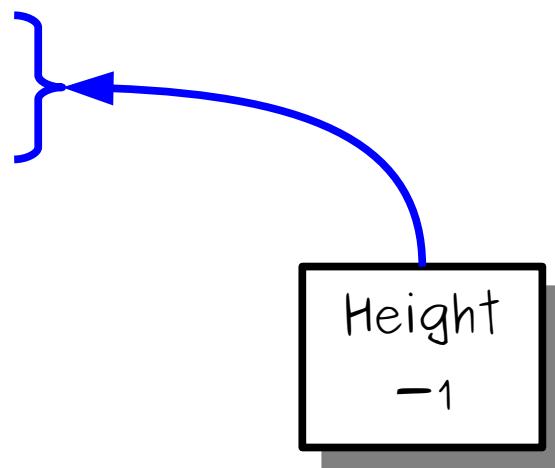
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Tree Terminology

- The **height** of a tree is the number of links in the longest path from the root to a leaf.
- By convention, an empty tree has height -1.



Building a BST

- First, draw the BST formed by inserting the values 1, 3, 5, 7, 2, 4, 6 into an empty tree.
- Then draw what you get if you insert the values 4, 6, 5, 2, 1, 7, and 3 into an empty tree.

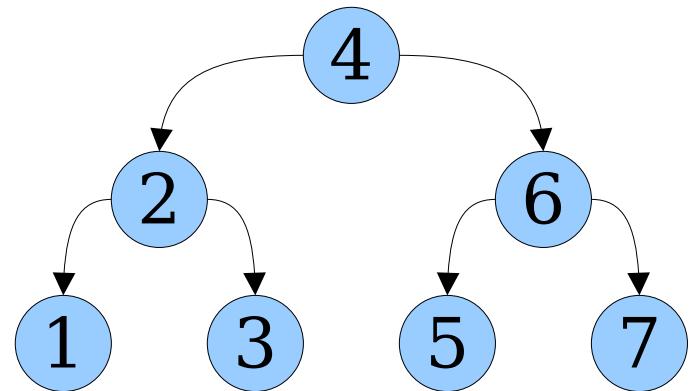
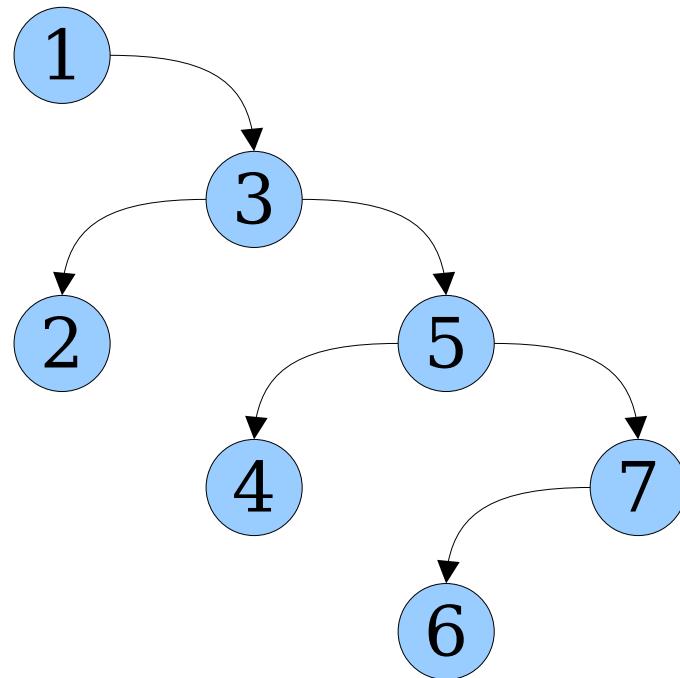
Draw these trees. What is the height of each tree?

Answer at

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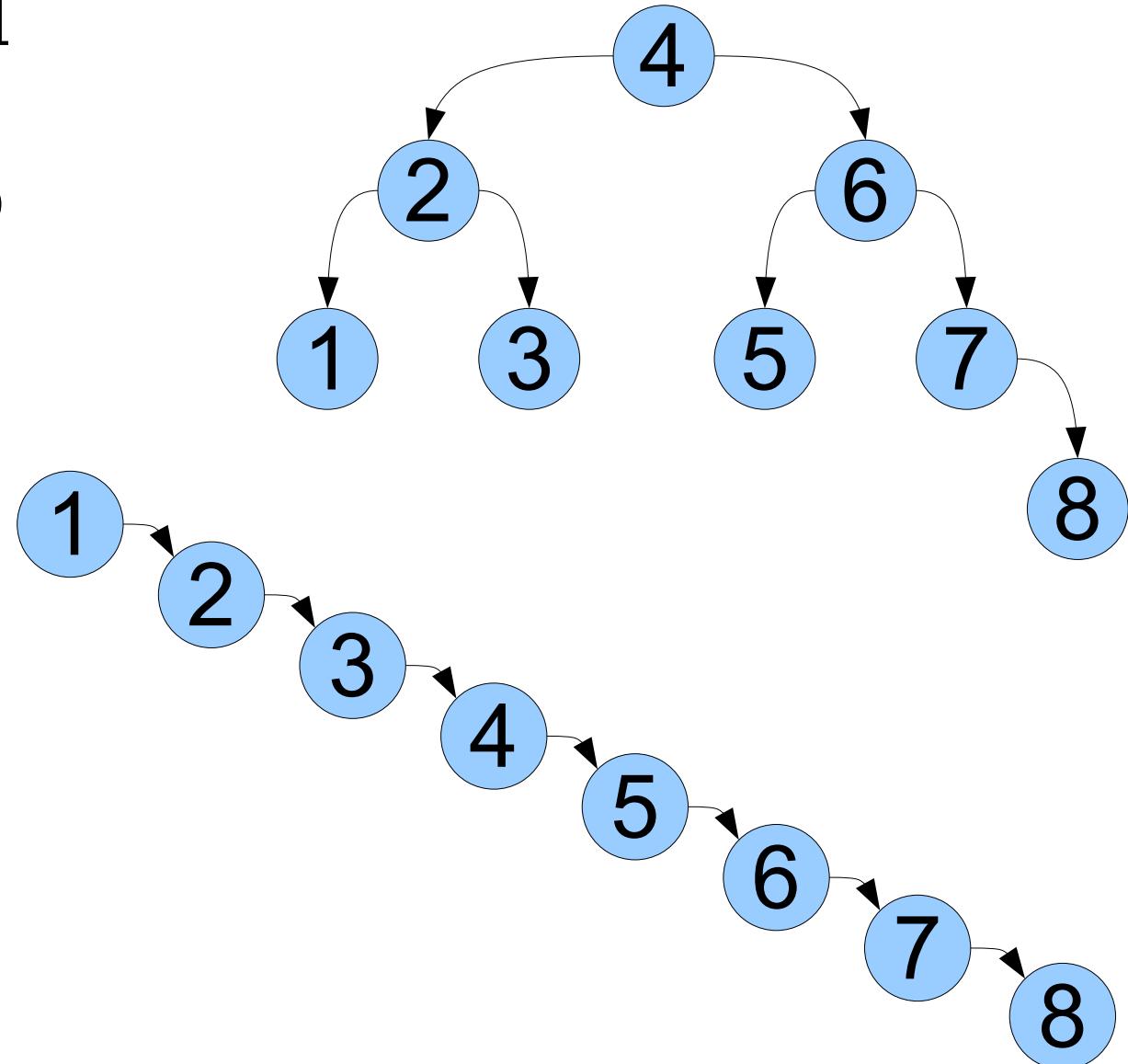
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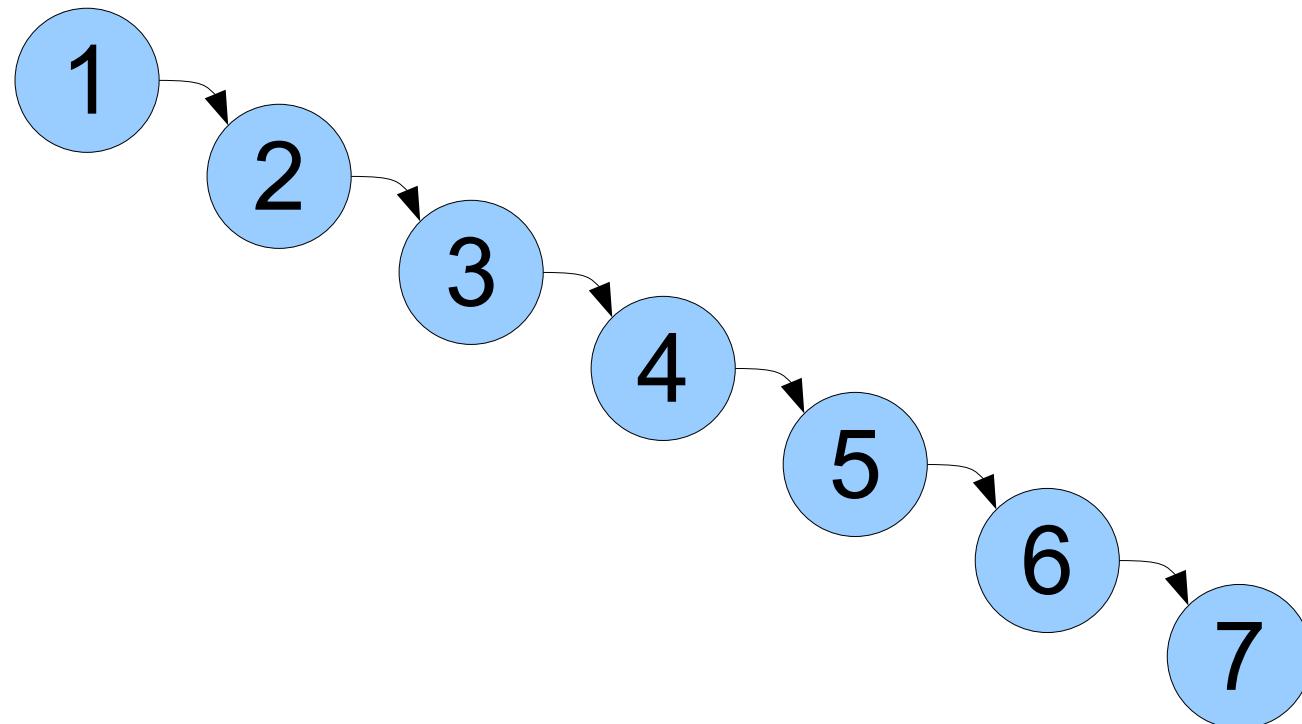
Efficiency Questions

- The time to add an element to a BST (or look up an element in a BST) depends on the height of the tree.
- The runtime is **$O(h)$** , where h is the height of the tree.



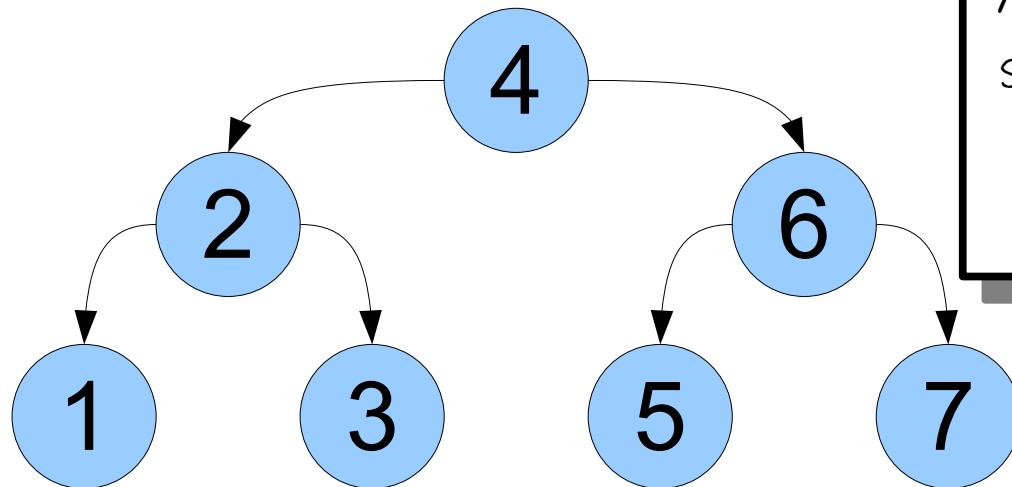
Tree Heights

- What is the maximum and minimum possible height of a tree with n nodes?
- Maximum height: all nodes in a chain. Height is $O(n)$.



Tree Heights

- What is the maximum and minimum possible height of a tree with n nodes?
- Maximum height: all nodes in a chain. Height is $O(n)$.
- Minimum height: tree is as complete as possible. Height is $O(\log n)$.



You can only double something $O(\log n)$ times before it exceeds n .

Balanced Trees

- A binary search tree is called ***balanced*** if its height is $O(\log n)$, where n is the number of nodes in the tree.
- Balanced trees are extremely efficient:
 - Lookups take time $O(\log n)$.
 - Insertions take time $O(\log n)$.
 - Deletions take time $O(\log n)$.
- ***Question:*** How do you balance a tree?

Balanced Trees

- A ***self-balancing tree*** is a BST that reshapes itself on insertions and deletions to stay balanced.
- There are many strategies for doing this. They're beautiful. They're clever. And they're beyond the scope of CS106B.
- Some suggested topics to read up on, if you're curious:
 - Red/black trees (take CS161 or CS166!)
 - AVL trees (covered in the textbook.)
 - Splay trees (trees that reshape on lookups.)
 - Scapegoat trees (yes, that's what they're called.)
 - Treaps (half binary heap, half binary search tree!)

What if you do no balancing at all?

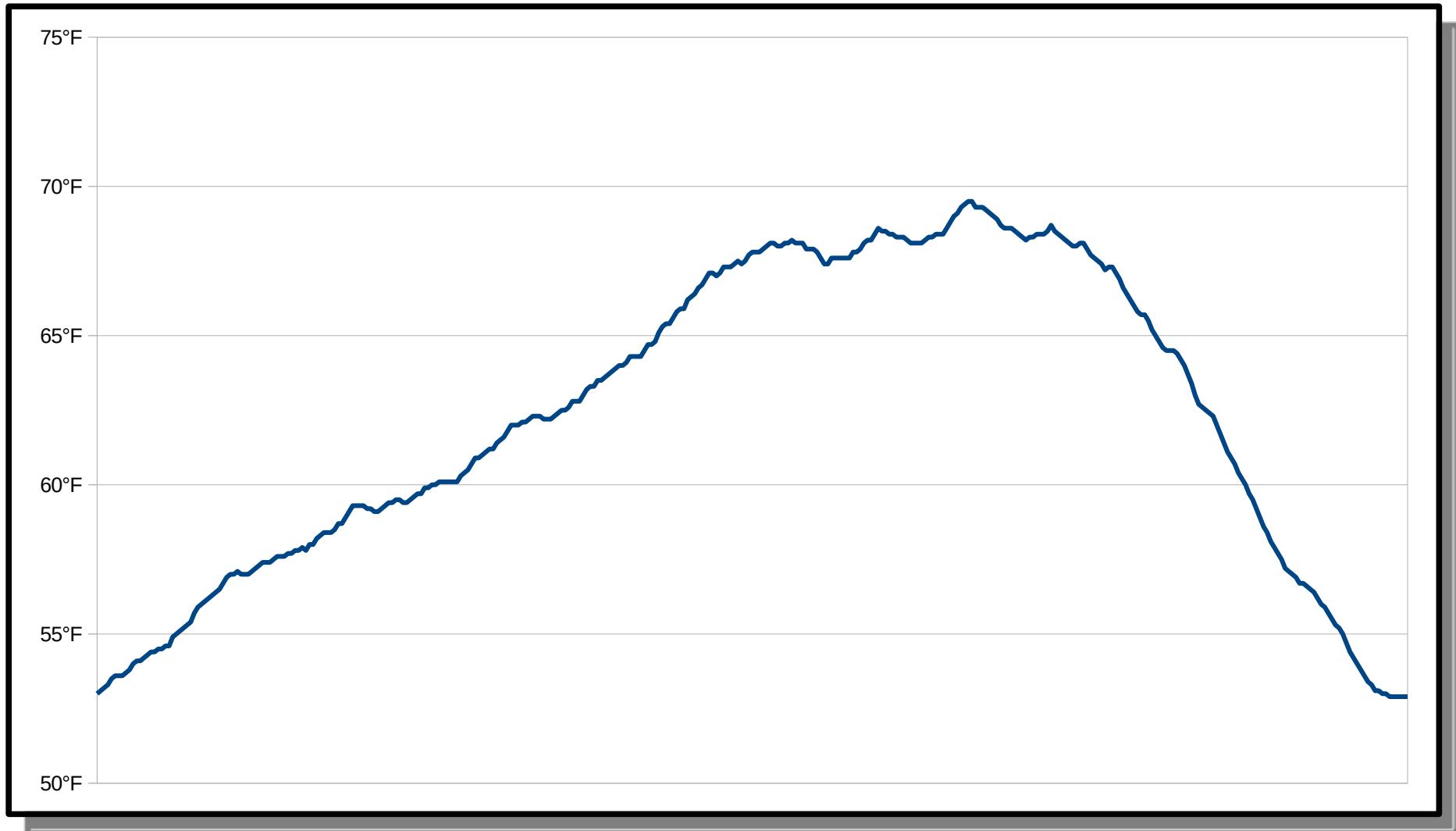
A Tale of Two Trees

- We have a thermometer that gives a temperature reading at 4PM each day. We insert the temperature readings into a BST each day, starting on January 1 and ending on December 31.
- There's a marathon race. We insert the names of the athletes into a BST as they cross the finish line.

Which BST will be more balanced?
Which BST will be less balanced?
Why?

Answer at

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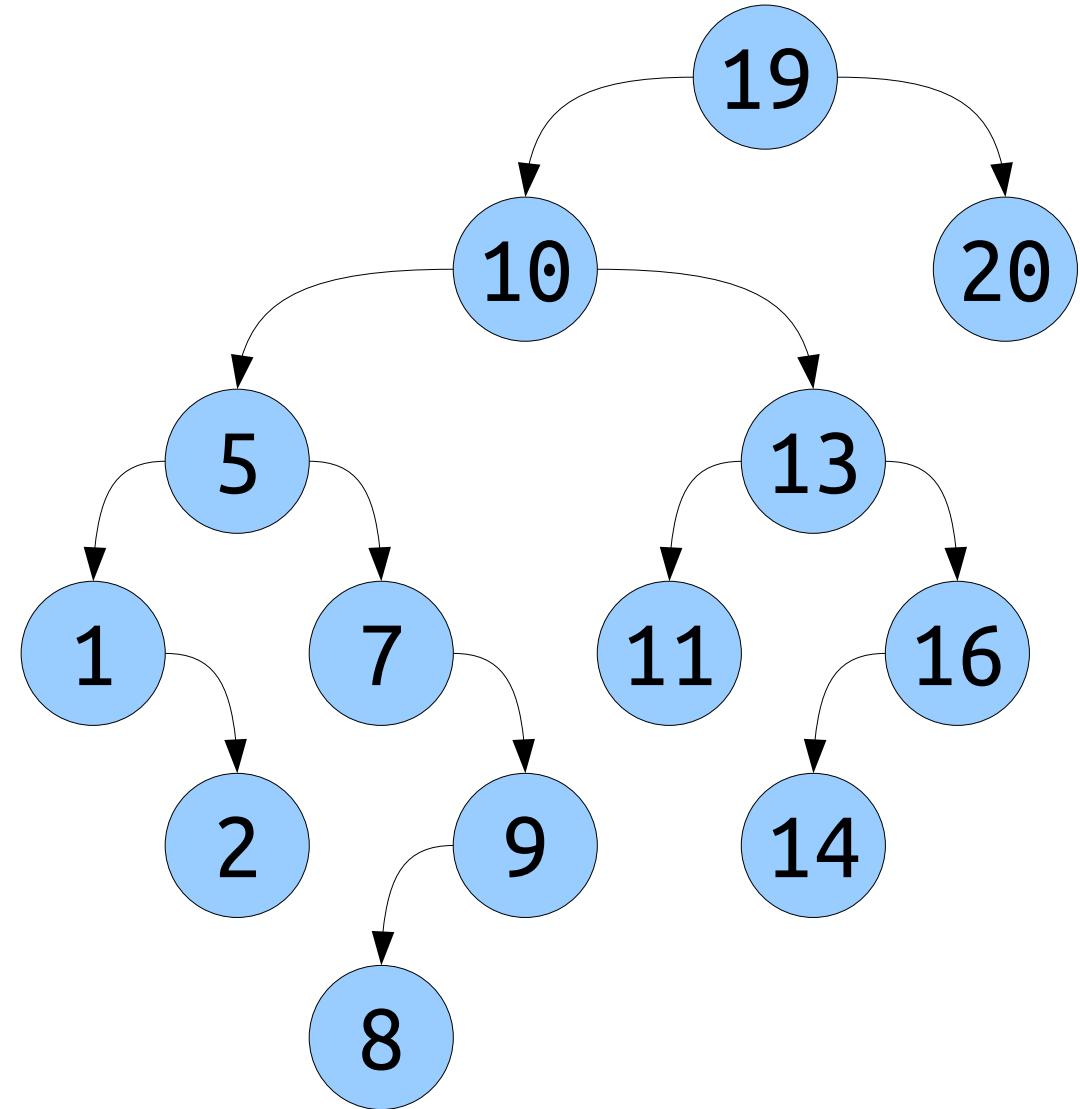


Temperature readings, inserted
daily at 4PM, from January 1 to
December 31.

(Data source: NOAA: SFO readings from Jan 1 - Dec 31 2010)

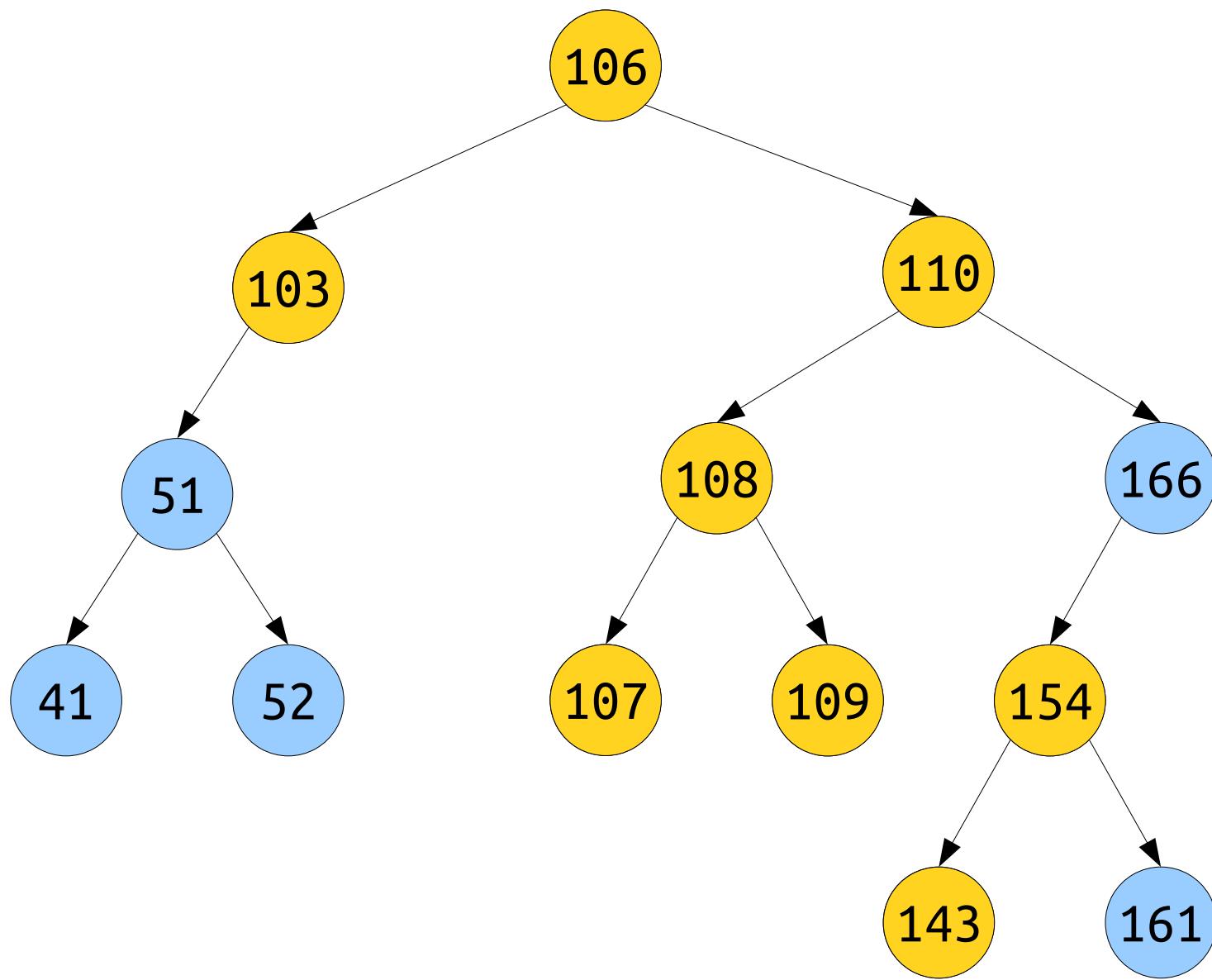
Balanced Trees

- **Theorem:** If you start with an empty tree and add in random values, then, with high probability, the tree is balanced.
- **Proof:** Take CS161!
- **Takeaway:** If you're adding elements to a BST and their values are actually random, then your tree is likely to be balanced.

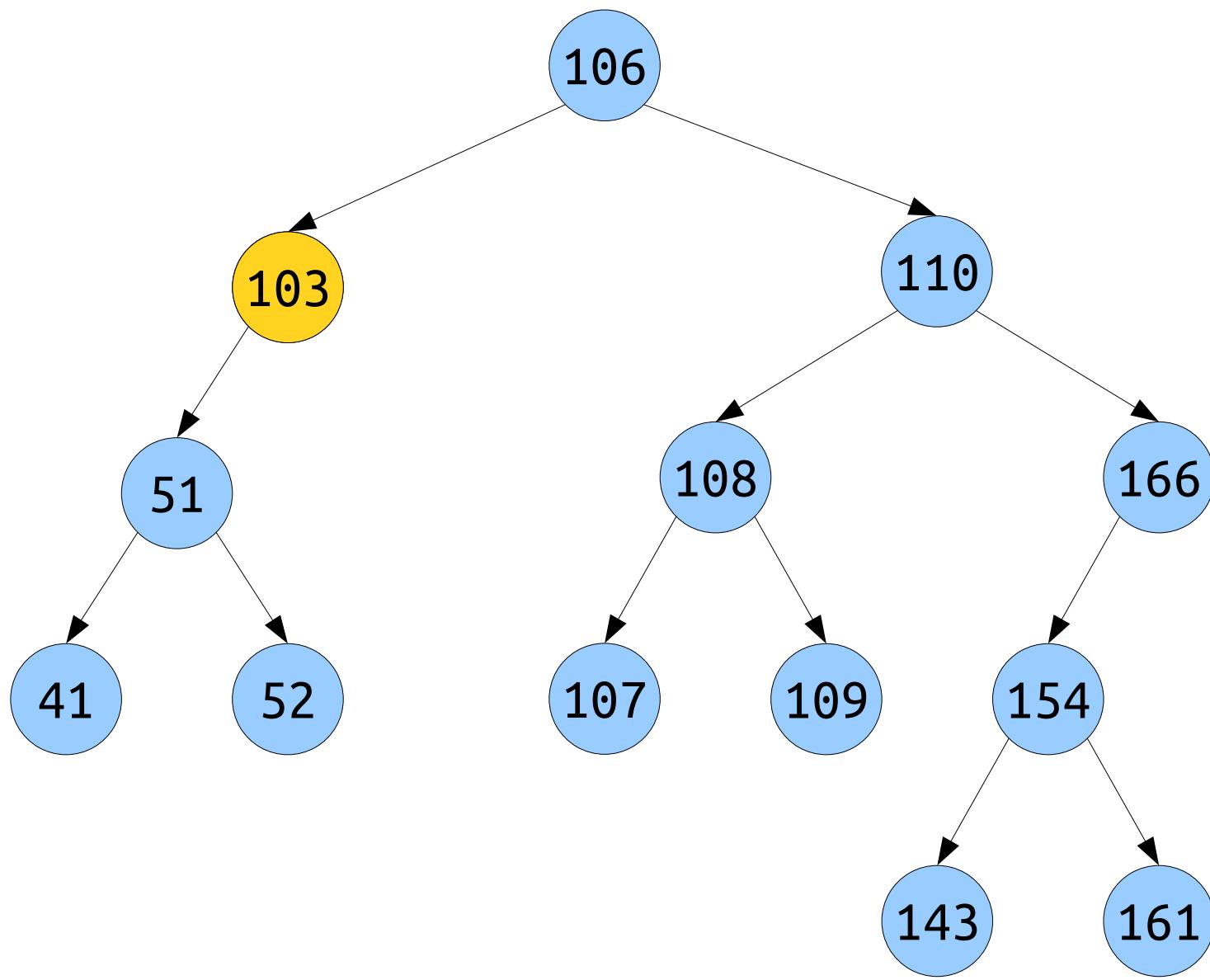


Range Searches

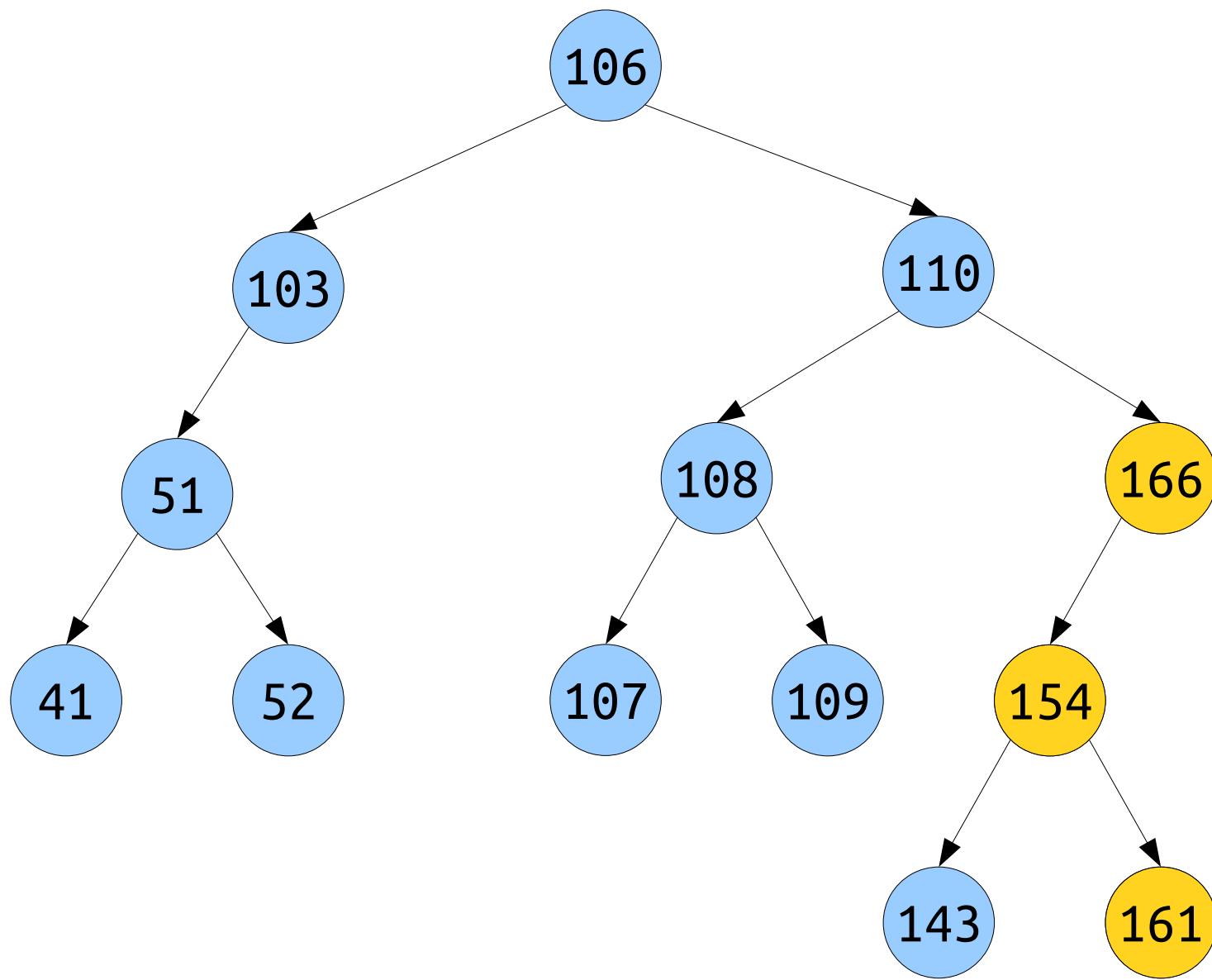




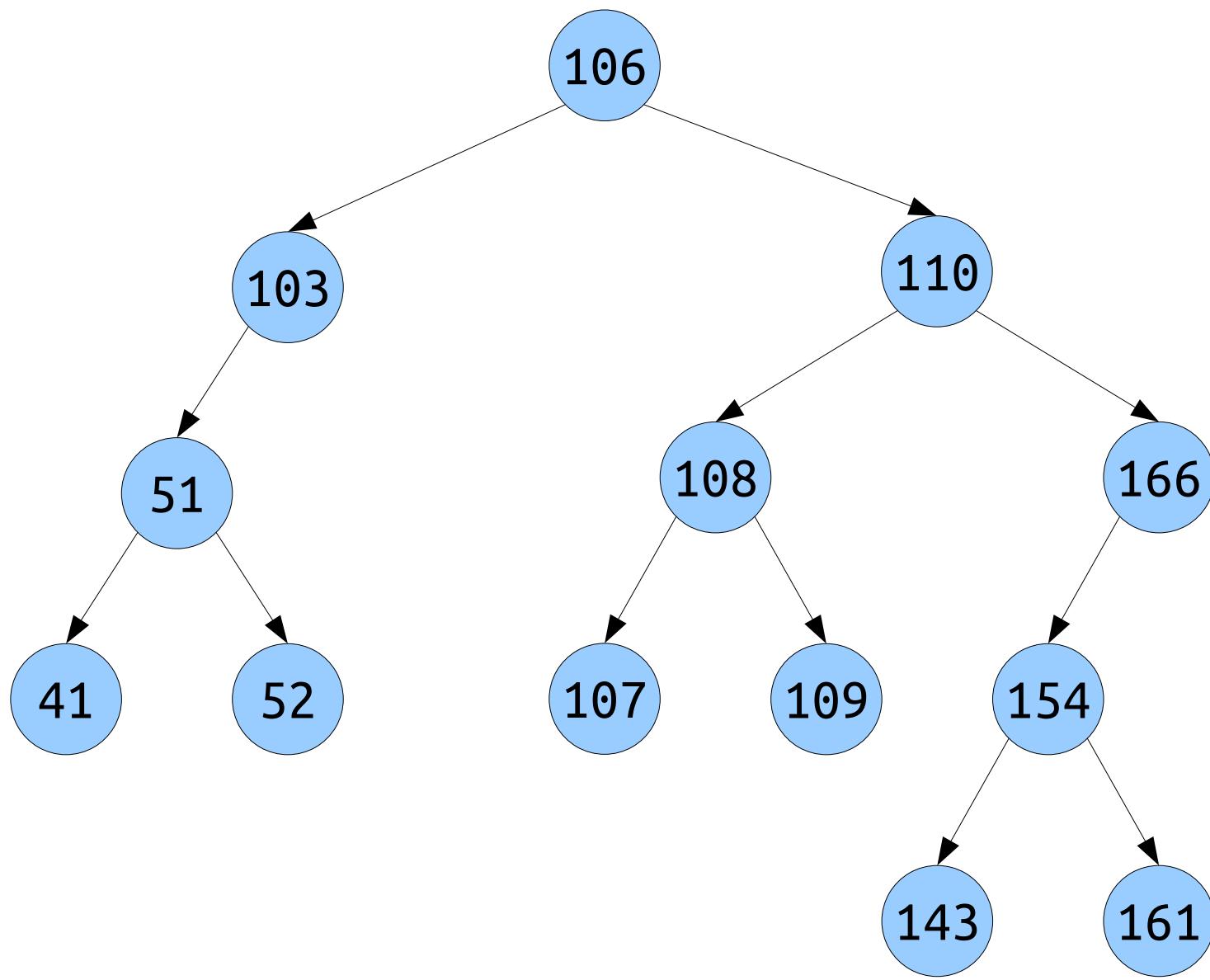
Find all elements in this tree in the range **[103, 154]**.



Find all elements in this tree in the range **[99, 105]**.



Find all elements in this tree in the range **[150, 170]**.



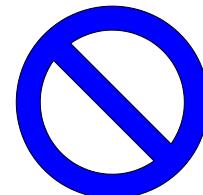
Find all elements in this tree in the range **[137, 138]**.

Range Searches

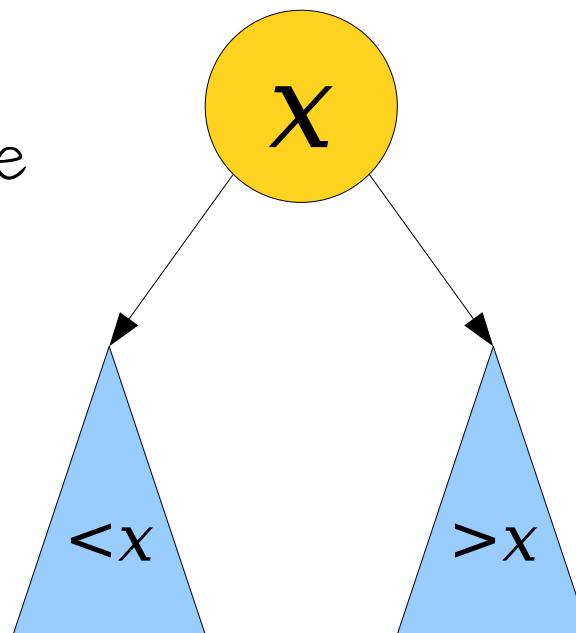
- We can use BSTs to do *range searches*, in which we find all values in the BST within some range.
- For example:
 - If the values in the BST are dates, we can find all events that occurred within some time window.
 - If the values in the BST are number of diagnostic scans ordered, we can find all doctors who order a disproportionate number of scans.

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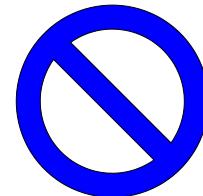
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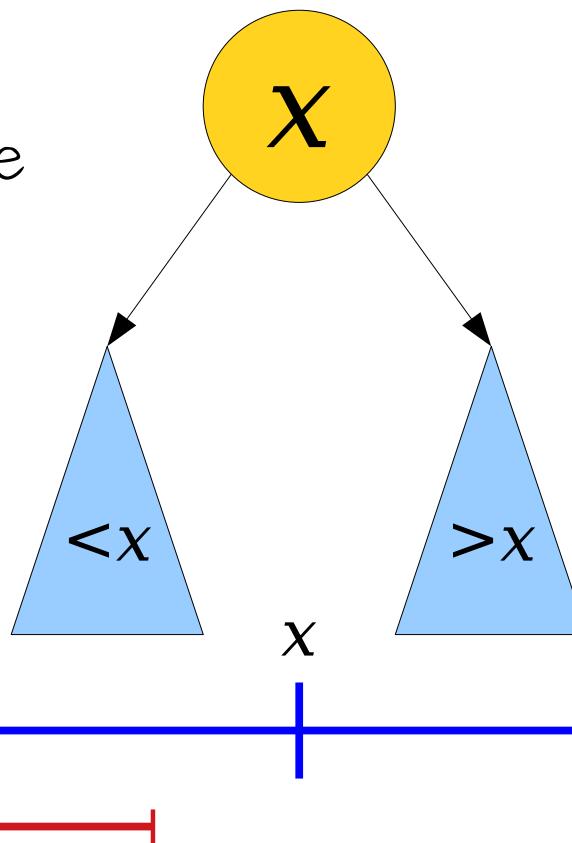
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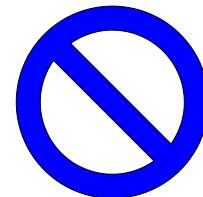
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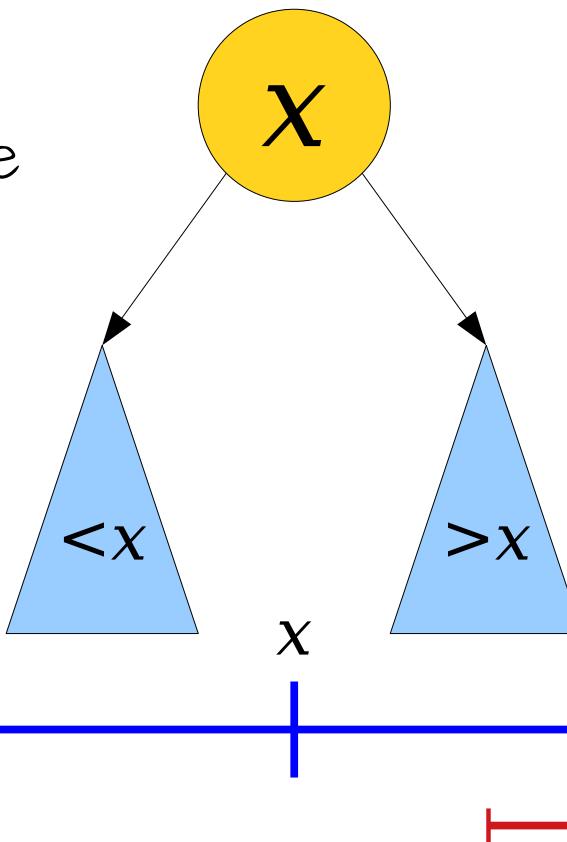
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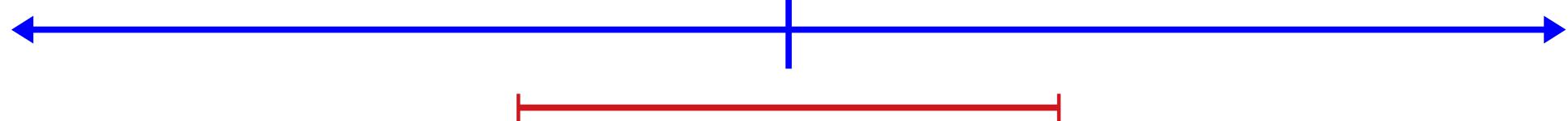
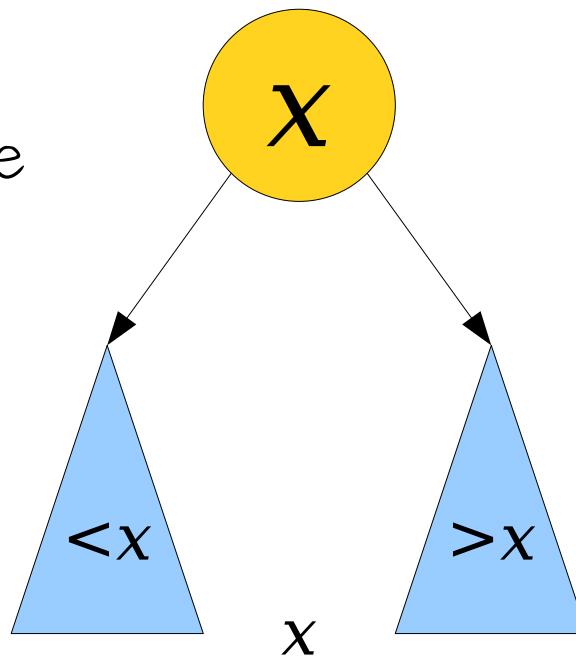
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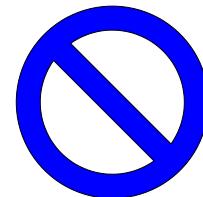
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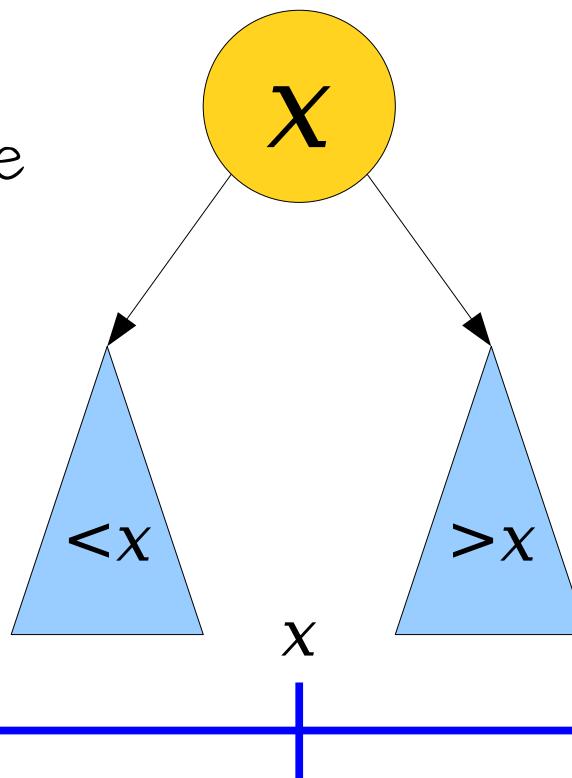
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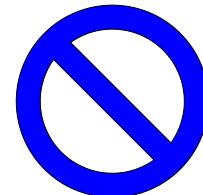
Range Searches

- A hybrid between an inorder traversal and a regular BST lookup!
- The idea:
 - If the node is in the range being searched, add it to the result.
 - Recursively explore each subtree that could potentially overlap with the range.
- ***Fun fact:*** The runtime of a range search is $O(h + z)$, where h is the height of the tree and z is the number of items in the range. Come chat with me after class if you're curious why this is!

To Summarize:

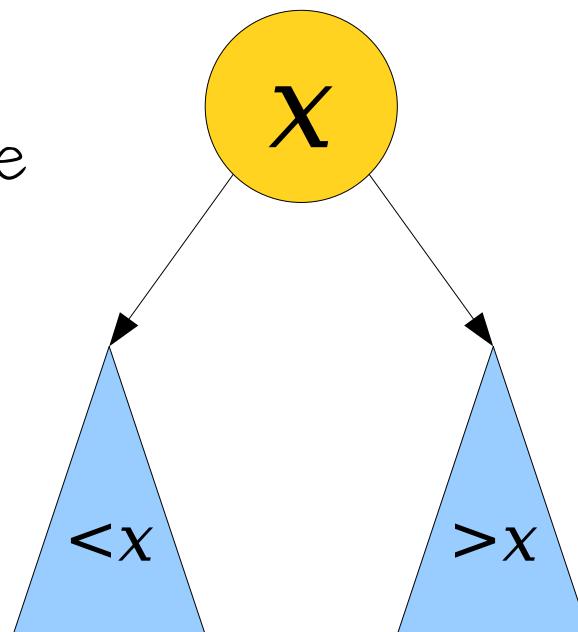
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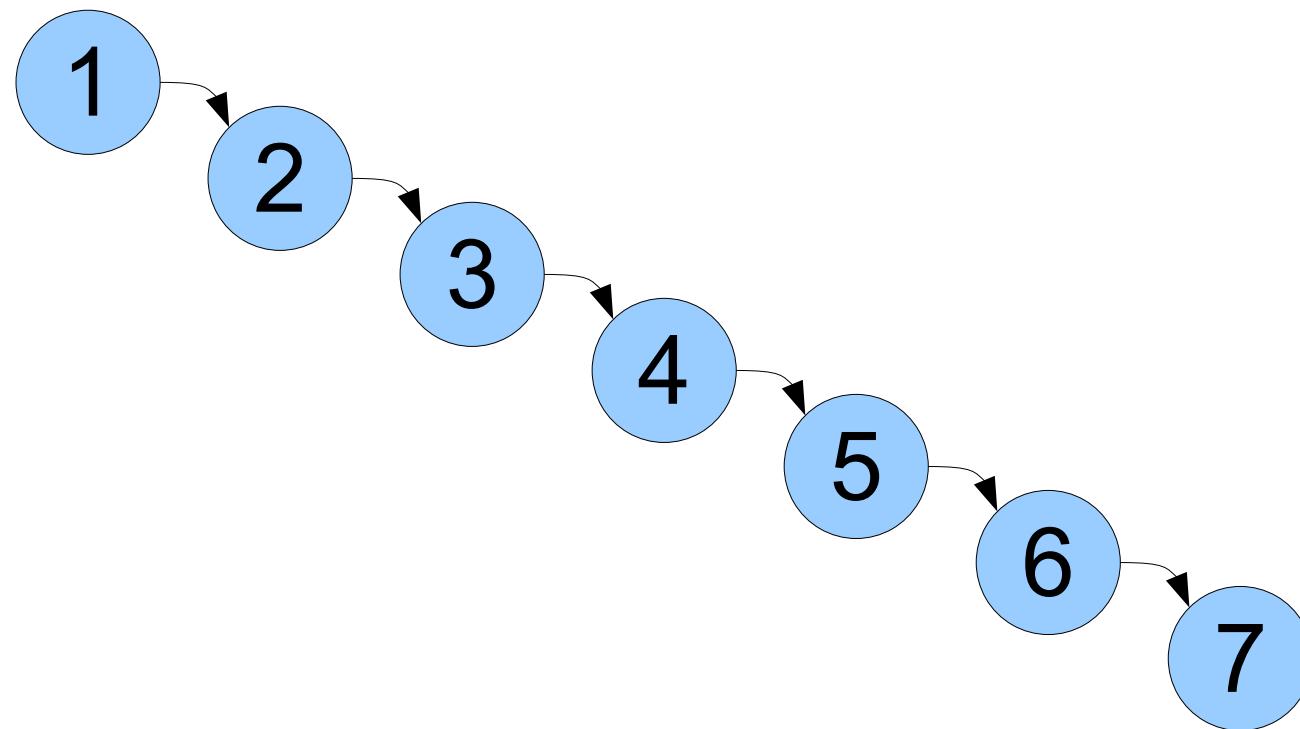
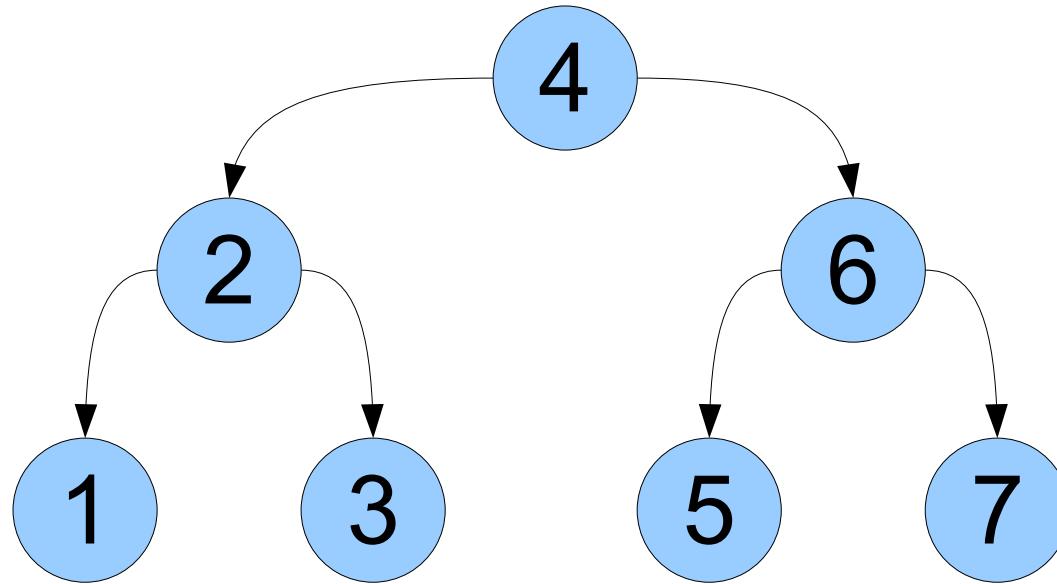


... and whose right
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of larger values.

```
struct Node {  
    Type value;  
    Node* left;    // Smaller values  
    Node* right;   // Bigger values  
};
```

```
bool contains(Node* root, const string& key) {
    if (root == nullptr) return false;
    else if (key == root->value) return true;
    else if (key < root->value) return contains(root->left, key);
    else return contains(root->right, key);
}

void insert(Node*& root, const string& key) {
    if (root == nullptr) {
        root = new Node;
        node->value = key;
        node->left = node->right = nullptr;
    } else if (key < root->value) {
        insert(root->left, key);
    } else if (key > root->value) {
        insert(root->right, key);
    } else {
        // Already here!
    }
}
```



```
void printContentsOf(Node* root) {  
    if (root == nullptr) return;  
  
    printContentsOf(root->left);  
    cout << root->value << endl;  
    printContentsOf(root->right);  
}  
  
void deleteTree(Node* root) {  
    if (root == nullptr) return;  
  
    deleteTree(root->left);  
    deleteTree(root->right);  
    delete root;  
}
```

```
void printInRange(Node* tree, const string& low, const string& high) {
    if (tree == nullptr) return;

    if (high < tree->value) {
        printInRange(tree->left, low, high);
    } else if (low > tree->value) {
        printInRange(tree->right, low, high);
    } else {
        printInRange(tree->left, low, high);
        cout << tree->value << endl;
        printInRange(tree->right, low, high);
    }
}
```

Your Action Items

- ***Read Chapter 16.1 - 16.2.***
 - All about BSTs!
- ***Work on Assignment 7.***
 - If you are following our timetable, you'll have finished the labyrinth and doubly-linked list warmups and should be in the middle of Particle Systems now.
 - Remember that you *can* use late days on this assignment and *cannot* use them on Assignment 8. Plan accordingly.
 - Don't use late days unnecessarily; that eats into your time for A8.
 - Don't save your late days "just in case" you need them on A8, since you can't use them there.
 - Need help? Have questions? Come talk to us in LaIR or during office hours.

Next Time

- ***Other Binary Trees***
 - BSTs are wonderful, but other tree structures with similar shapes exist.
- ***Huffman Coding***
 - Practical data compression – with trees!